# Test 1

Total mark
20

## 1 Choose the correct answer from those given : (12 marks)

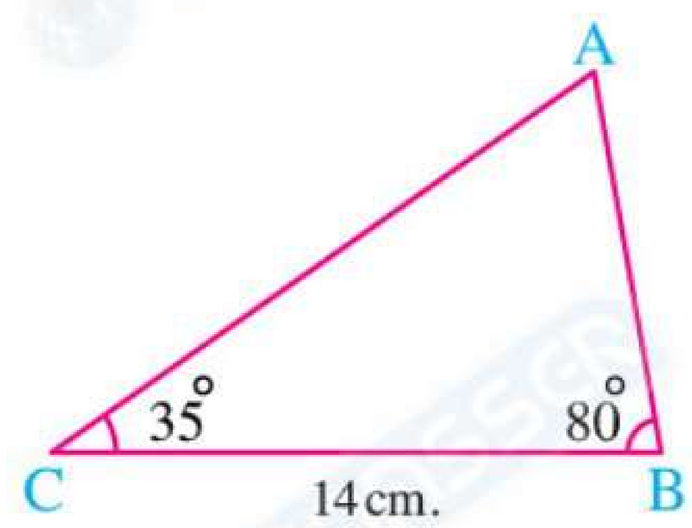
1. $\lim\limits_{x \to 3} \dfrac{x^2 - 5x + 6}{x^2 - 9} = \dots\dots$
   (a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{5}$ (d) $\frac{1}{4}$

2. The domain of the function $f : f(x) = \dfrac{\sqrt{x - 2}}{x - 4}$ is $\dots\dots$
   (a) $[2, \infty[$ (b) $]-\infty, 2]$ (c) $[2, \infty[ - \{4\}$ (d) $]-\infty, 2] - \{4\}$

3. All the following functions is one - to - one on its domain except $f(x) = \dots\dots$
   (a) $3x$ (b) $\frac{1}{x}$ (c) $5$ (d) $x^3$

4. If the radius of the circumcircle of triangle ABC equals 3 cm.
   and $\sin A + \sin B + \sin C = 2$ , then the perimeter of the triangle = $\dots\dots$ cm.
   (a) 6 (b) 9 (c) 12 (d) 24

5. The greatest side in the given triangle $\simeq \dots\dots$ cm.
   (to the nearest integer)
   (a) 20 (b) 16
   (c) 15 (d) 14




6. If $\lim\limits_{x \to 1} \dfrac{x^2 - k}{x + 2} = -1$ , then $k = \dots\dots$
   (a) 4 (b) $-2$ (c) 2 (d) $\pm 2$

7. The function $f : f(x) = |x + 2|$ is decreasing on the interval $\dots\dots$
   (a) $]0, \infty[$ (b) $[2, \infty[$ (c) $]-2, \infty[$ (d) $]-\infty, -2[$

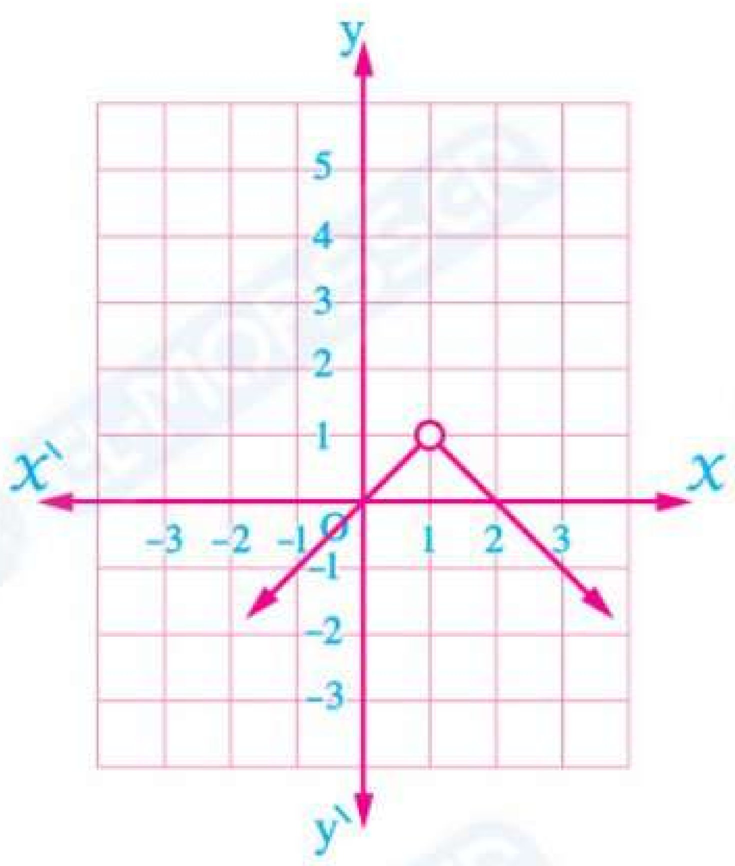
8. If $f$ is an odd function , then $\dfrac{7f(3) + 3f(-3)}{2f(-3)} = \dots\dots$
   (a) 2 (b) $-2$ (c) 5 (d) $-0.5$

9. If $f(x) = x - 3$ , $g(x) = x^2$ , then $(f \circ g)(5) = \dots\dots$
   (a) 2 (b) 4 (c) 22 (d) 25

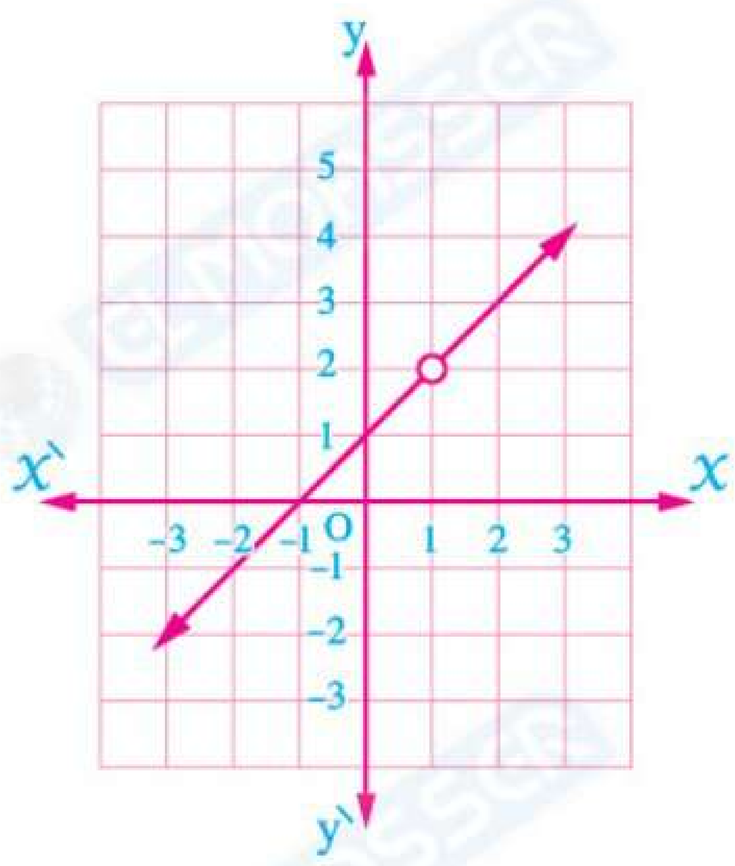
10. The symmetric point on the curve of the function $f : f(x) = \dfrac{x + 1}{x}$ is $\dots\dots$
    (a) $(0, 1)$ (b) $(1, 0)$ (c) $(0, 0)$ (d) $(1, -1)$

11. $\lim\limits_{x \to 4} \dfrac{\sqrt{x} - 2}{x - 4} = \dots\dots$
    (a) 2 (b) 4 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

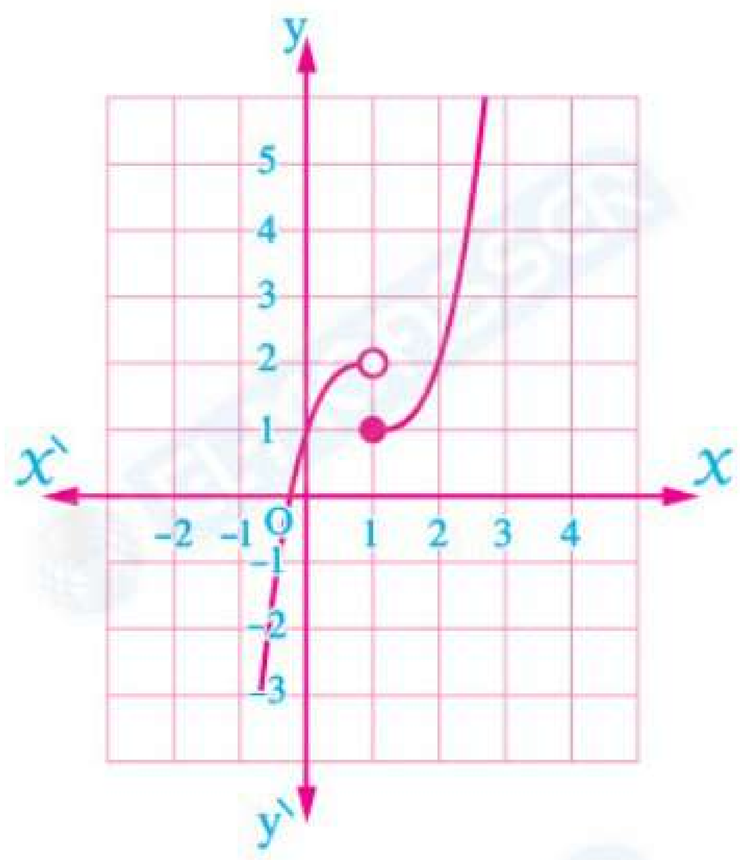
**12** Which of the following functions has no limit at $x = 1$ ?



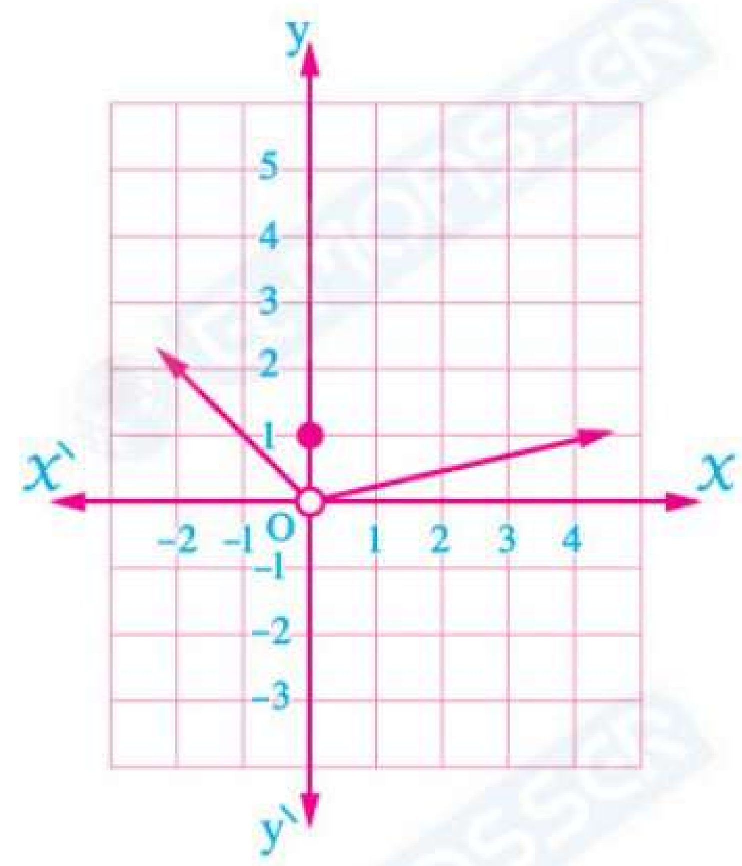

(a)




(b)




(c)




(d)

## 2 Answer the following questions :

**1** Graph the function $f : f(x) = \begin{cases} x^2 + 1 & , x > 0 \\ -x^2 - 1 & , x < 0 \end{cases}$

from the graph determine the range of the function and discuss its monotony.

(2 marks)

**2** If $f(x) = x^2 - 3$ , $g(x) = \sqrt{x - 2}$ , find $(f \circ g)(x)$ in the simplest form , determining its domain , then find $(f \circ g)(3)$ (2 marks)

**3** Find : $\lim\limits_{x \to 1} \dfrac{\sqrt{4x + 5} - 3}{x - 1}$ (2 marks)

**4** ABCD is a parallelogram in which m (∠ A) = 50° , m (∠ DBC) = 70° and BD = 8 cm.

Find the perimeter of the parallelogram to the nearest whole number. (2 marks)

# Test 2

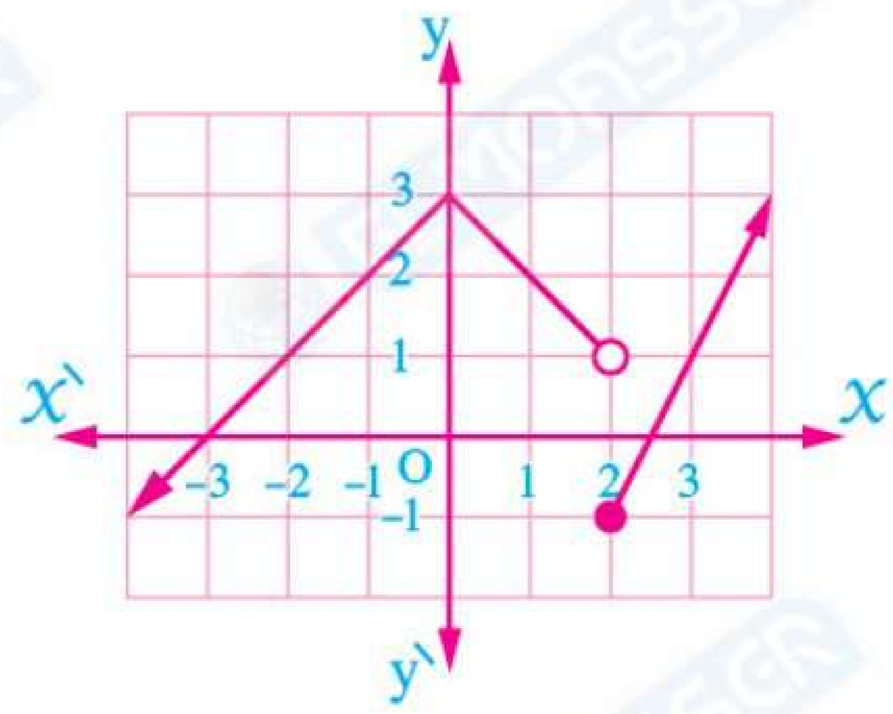
Total mark 20

## 1 Choose the correct answer from those given : (12 marks)

1 In $\Delta ABC$ , $\frac{4\,b}{\sin B} =$ .............. r , where r is the radius of the circle passes through the vertices of triangle ABC.

(a) 4 (b) 8 (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

2 The opposite figure represents the curve of the function $f$ , then $\lim_{x \to 2} f(x) =$ ..............



(a) 3

(b) 1

(c) – 1

(d) not exist.

3 The one - to - one function from the functions defind by the following rules is ..............

(a) $f(x) = x^2$ (b) $f(x) = -2$ (c) $f(x) = |x|$ (d) $f(x) = \frac{1}{x}$

4 If $f$ is an even function and the curve of the function passes through the point $(-3 , 2m + 1)$ and $f(3) = 5$ , then m = ..............

(a) zero (b) – 1 (c) 1 (d) 2

5 The domain of the function $f : f(x) = \sqrt{x - 3}$ is ..............

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $[3 , \infty[$ (d) $]-\infty , 3[$

6 If $f(x) = \sqrt{x}$ , $g(x) = x^2 - 1$ , then $(f \circ g)(-3) =$ ..............

(a) 2 (b) – 4 (c) $2\sqrt{2}$ (d) undefined

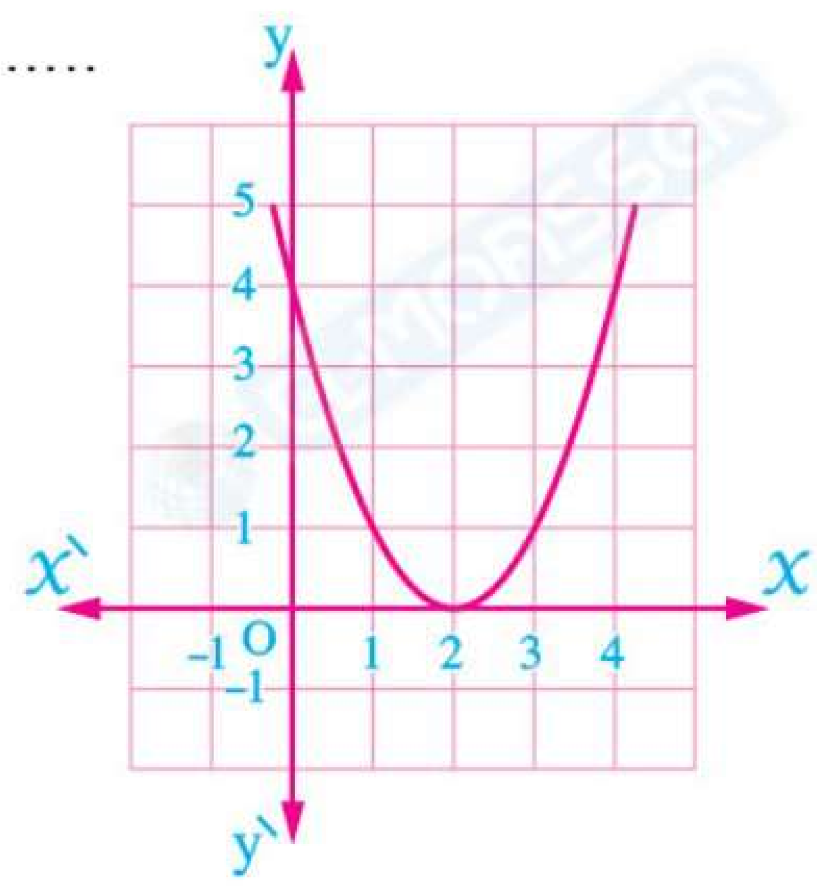
7 $\lim_{x \to 7} \frac{x^2 - 49}{7 - x} =$ ..............

(a) 14 (b) – 49 (c) 49 (d) – 14

8 The point of symmetry of the function $f : f(x) = (x - 2)^3 + 1$ is ..............

(a) (2 , 1) (b) (2 , – 1) (c) (– 2 , 1) (d) (– 1 , – 2)

9 The opposite figure represents the function $f : f(x) =$ .............. where $f : \mathbb{R} \longrightarrow \mathbb{R}$



(a) $x^2 + 2$ (b) $(x + 2)^2$

(c) $x^2 - 4x + 4$ (d) $-(x - 2)^2$

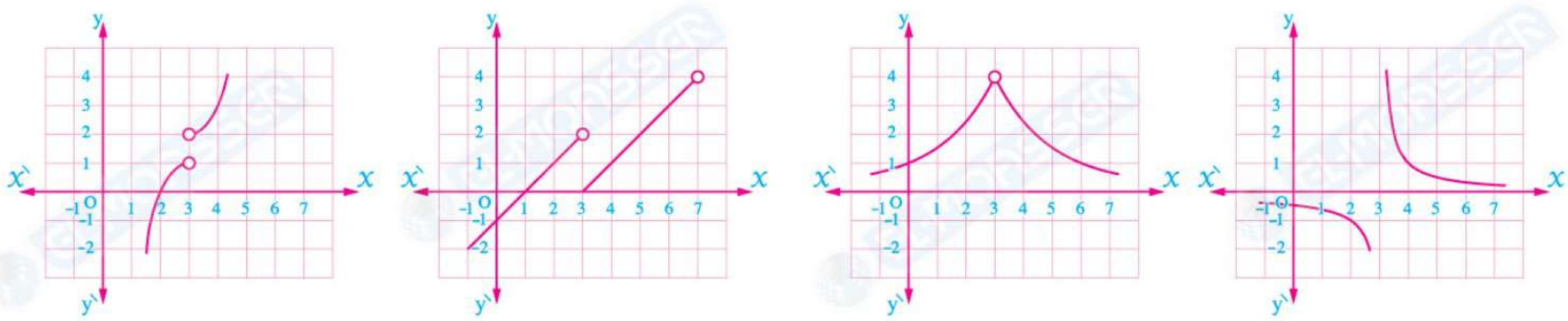
10 If $\lim\limits_{x \to 3} \dfrac{x^2 - \ell}{x - 3} = m$ , then $(\ell , m) =$ ...............

ⓐ (3 , 3)　　ⓑ (– 9 , 0)　　ⓒ (9 , 6)　　ⓓ (0 , 0)

11 In ΔXYZ , if m (∠ X) : m (∠ Y) : m (∠ Z) = 2 : 3 : 1 , then $x : y : z =$ ...............

ⓐ $3 : \sqrt{2} : 1$　　ⓑ $\sqrt{3} : 2 : 1$　　ⓒ $\sqrt{2} : \sqrt{3} : 1$　　ⓓ $2 : 3 : 1$

12 Which of the following functions has a limit at $x = 3$ ?



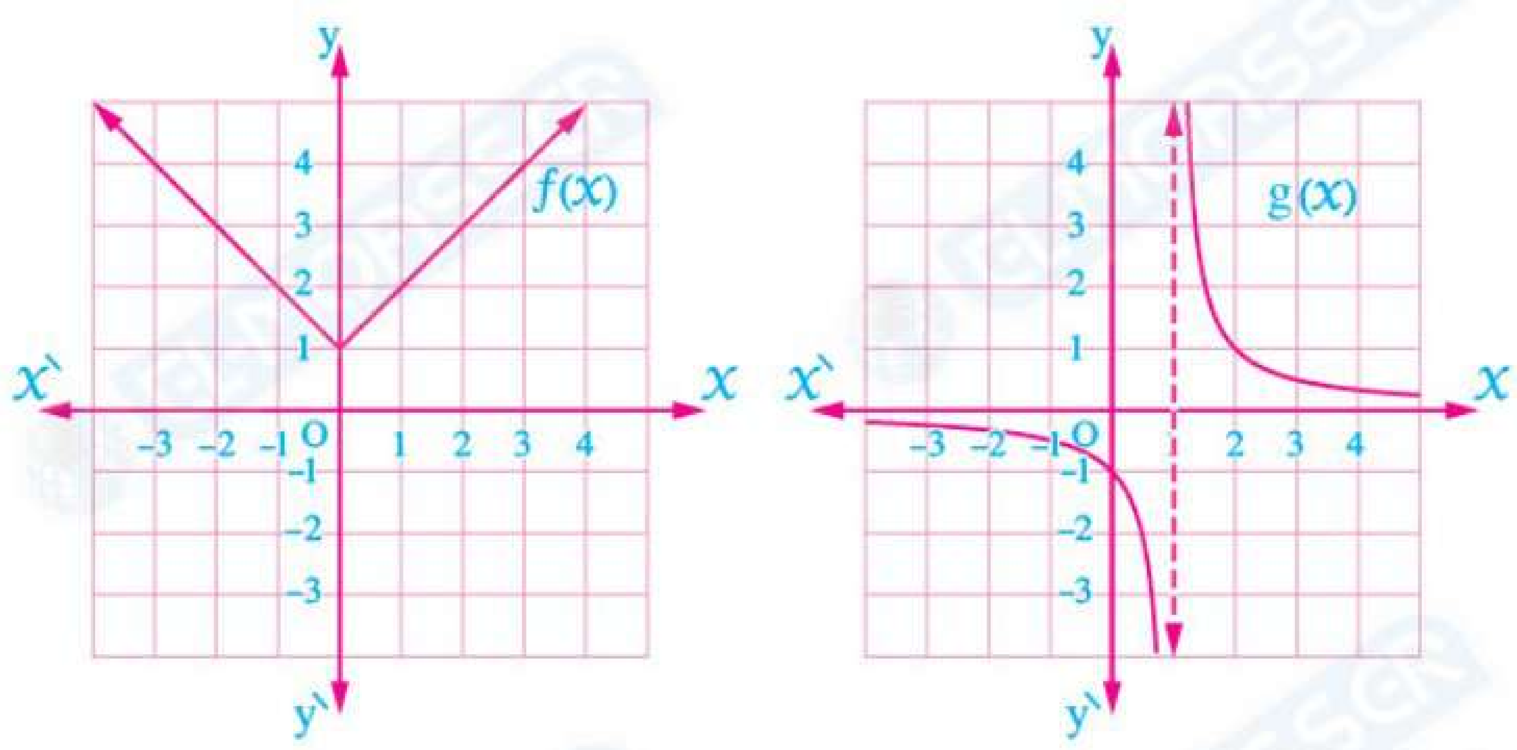
ⓐ　　ⓑ　　ⓒ　　ⓓ

## 2 Answer the following questions :

1 Graph the curve of the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ , $f(x) = |x| + 1$ From the graph find the range and discuss the monotony and determine its type wheather it is odd , even or neither. (2 marks)

2 **In the opposite figure :**

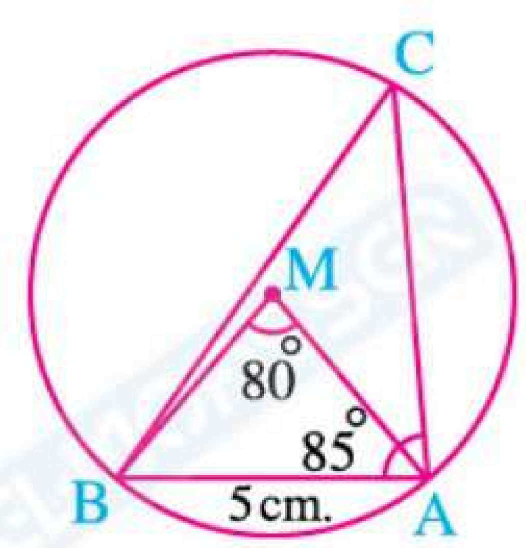
Find $(f \circ g)(x)$ and state its domain




(2 marks)

3 **Find :** $\lim\limits_{x \to 5} \dfrac{\sqrt{x + 11} - 4}{x^2 - 25}$ (2 marks)

4 **In the opposite figure :**




A circle M , AB = 5 cm. , m (∠ AMB) = 80°

and m (∠ CAB) = 85°

**Find : ( 1 )** The perimeter of Δ ABC.

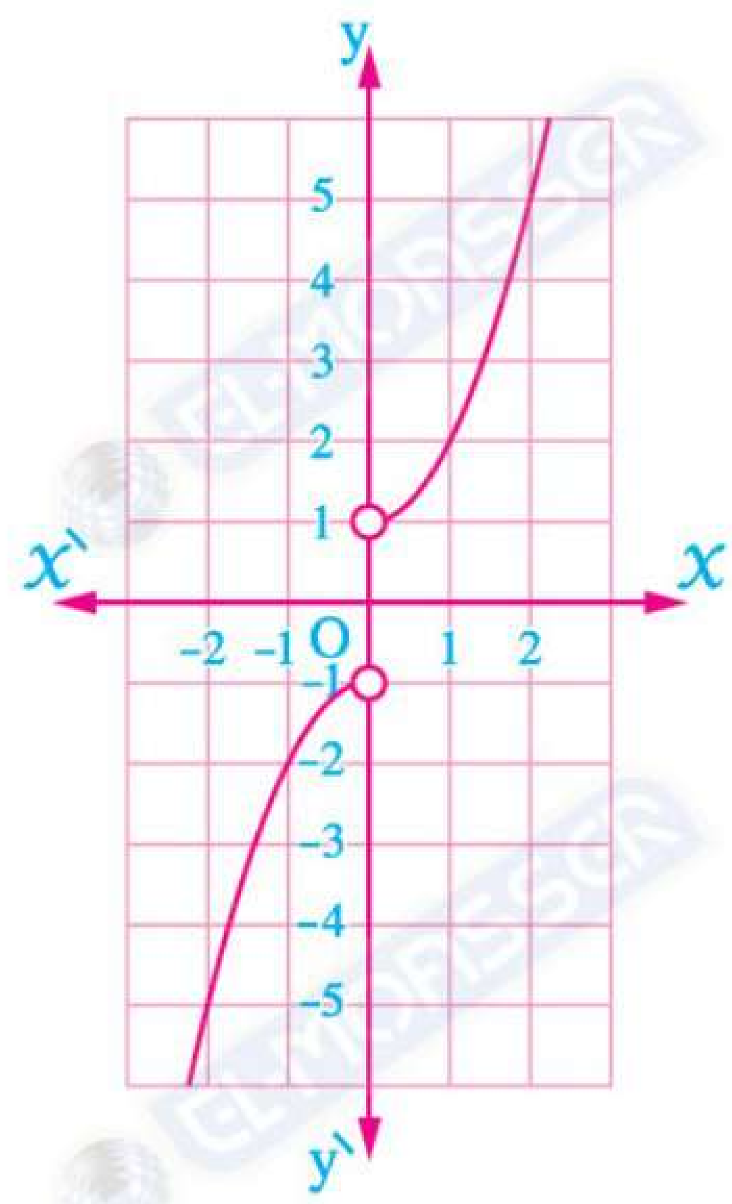
**( 2 )** The area of the circle M. (2 marks)

# Answers of Test 1

**1** 1 (b) 2 (c) 3 (c) 4 (c) 5 (c) 6 (a) 7 (d) 8 (b)
9 (c) 10 (a) 11 (d) 12 (c)

**2** (1) From the graph :

- The range of the function $f = \mathbb{R} - [-1 , 1]$
- The function is increasing on $\mathbb{R} - \{0\}$




(2) $\because (f \circ g)(x) = f(g(x)) = f(\sqrt{x-2}) = (\sqrt{x-2})^2 - 3 = x - 2 - 3 = x - 5$

$, \because D_1 = \text{domain of } g = [2 , \infty[$

, the values of $x$ which makes g $(x)$ in the domain of $f$ is $D_2 = \mathbb{R}$

$\therefore$ The domain $(f \circ g) = D_1 \cap D_2 = [2 , \infty[$ , $(f \circ g)(3) = 3 - 5 = -2$

(3) $\lim\limits_{x \to 1} \frac{\sqrt{4x+5} - 3}{x - 1} \times \frac{\sqrt{4x+5} + 3}{\sqrt{4x+5} + 3} = \lim\limits_{x \to 1} \frac{4x + 5 - 9}{(x-1)(\sqrt{4x+5} + 3)}$

$= \lim\limits_{x \to 1} \frac{4\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{4x+5} + 3)} = \frac{4}{6} = \frac{2}{3}$

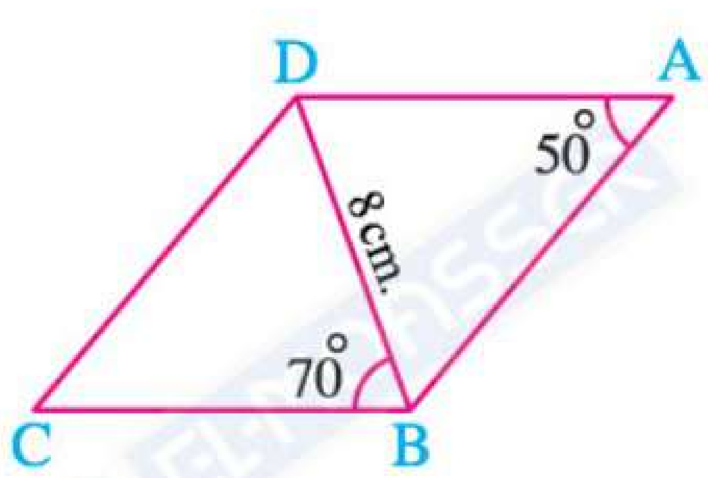
(4) $\because$ ABCD is a parallelogram $\therefore m(\angle C) = 50°$

In $\Delta$ BDC : $m(\angle BDC) = 180° - (50° + 70°) = 60°$

$\therefore \frac{8}{\sin 50°} = \frac{BC}{\sin 60°} = \frac{DC}{\sin 70°}$

$\therefore BC \simeq 9$ cm. , $DC \simeq 9.8$ cm.

$\therefore$ The perimeter of the parallelogram $= 2(BC + CD) \simeq 38$ cm.

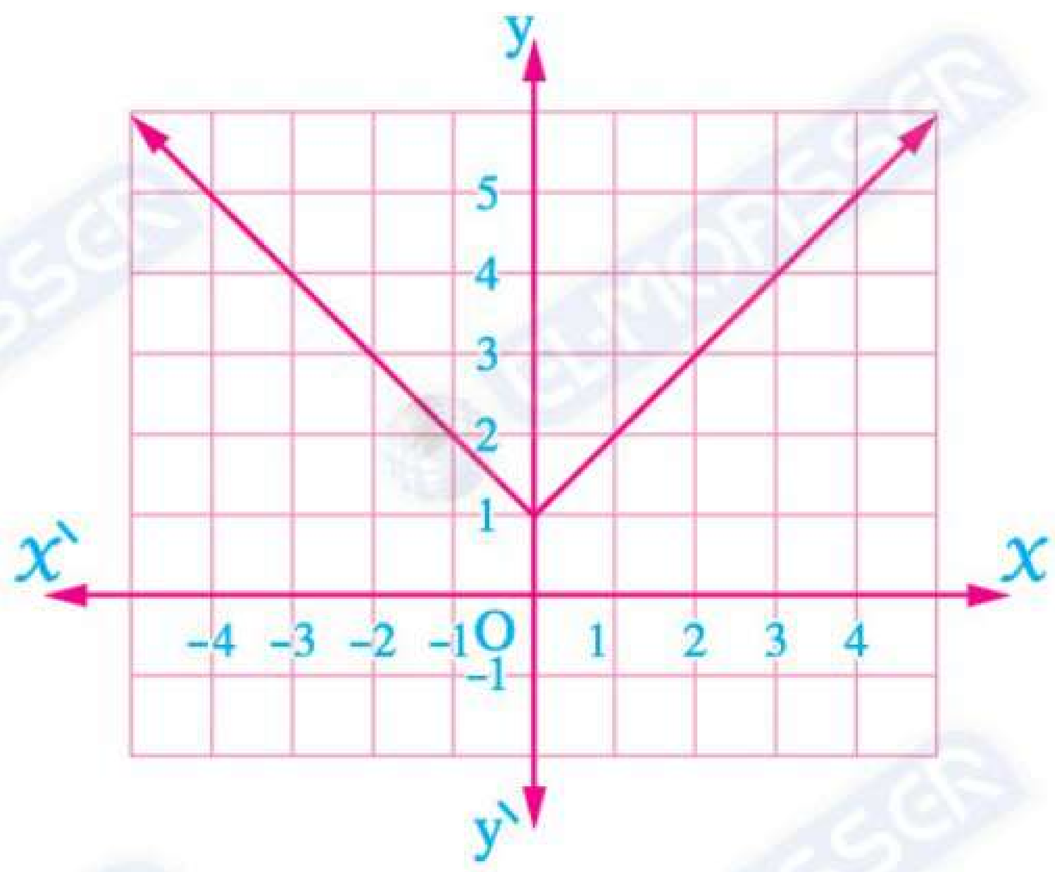


# Answers of Test 2

**1**

1 (b) 2 (d) 3 (d) 4 (d) 5 (c) 6 (c) 7 (d)
8 (a) 9 (c) 10 (c) 11 (b) 12 (c)

**2** (1) From the figure :




- The range = $[1, \infty[$
- The monotony :
  The function is decreasing on $]-\infty, 0[$
  and increasing on $]0, \infty[$
- The type : Even.

(2) $f(x) = |x| + 1$ , $g(x) = \frac{1}{x-1}$

$\therefore (f \circ g)(x) = f[g(x)] = f\left(\frac{1}{x-1}\right) = \left|\frac{1}{x-1}\right| + 1$

$\therefore D_1$ = domain of g $= \mathbb{R} - \{1\}$

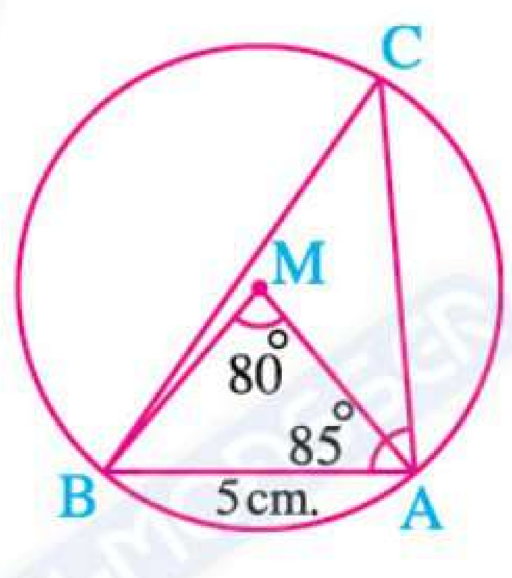
, the values of $x$ which makes g ($x$) in the domain of $f$ is $D_2 = \mathbb{R}$

$\therefore$ Domain of $(f \circ g) = D_1 \cap D_2 = \mathbb{R} - \{1\}$

(3) $\lim_{x \to 5} \frac{\sqrt{x+11} - 4}{x^2 - 25} \times \frac{\sqrt{x+11} + 4}{\sqrt{x+11} + 4} = \lim_{x \to 5} \frac{x + 11 - 16}{(x-5)(x+5)(\sqrt{x+11} + 4)}$

$= \lim_{x \to 5} \frac{1}{(x+5)(\sqrt{x+11} + 4)} = \frac{1}{80}$

(4) In $\Delta$ ABC : m ($\angle$ C) = $\frac{1}{2}$ m ($\angle$ AMB) = 40°




$\therefore$ m ($\angle$ ABC) = 180° – (40° + 85°) = 55°

$\therefore \frac{5}{\sin 40°} = \frac{b}{\sin 55°} = \frac{a}{\sin 85°} = 2r = \frac{\text{Perimeter of } \Delta ABC}{\sin 40° + \sin 55° + \sin 85°}$

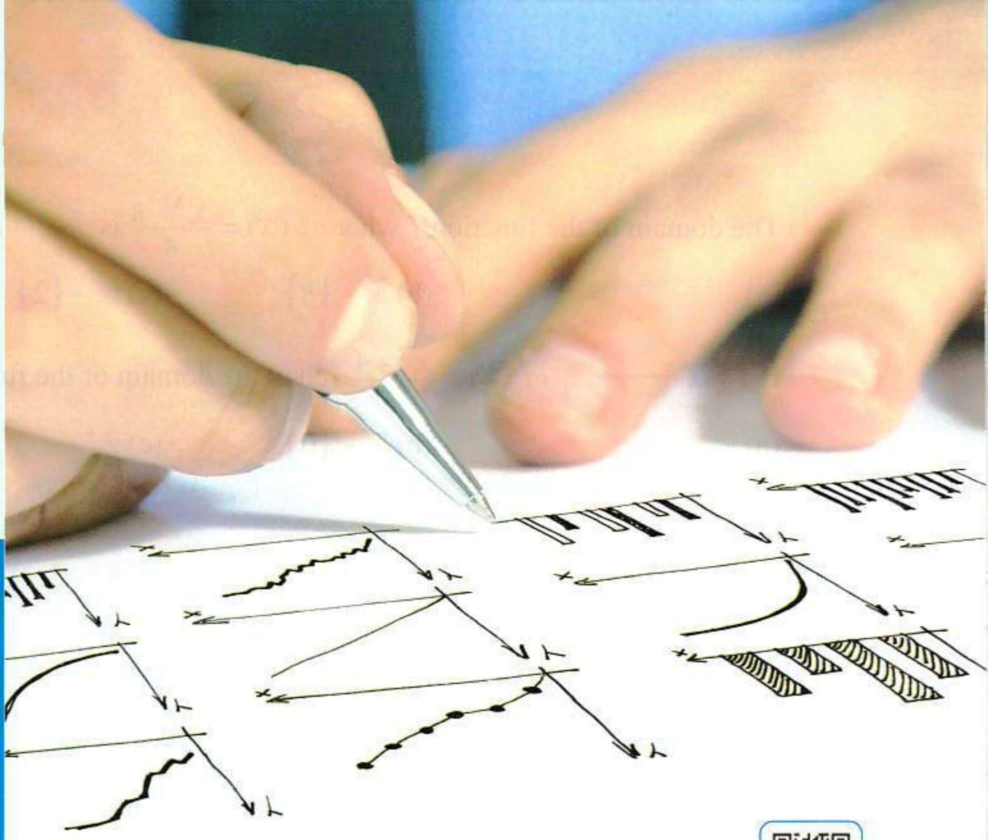
$\therefore$ Perimeter of $\Delta$ ABC $\simeq$ 19.12 cm.

, r = 3.89 cm.

$\therefore$ The area of the circle M $= \pi r^2 = \pi (3.89)^2 \simeq 47.5 \text{ cm}^2$

## Exercise 1

# Real functions



From the school book ● Understand ○ Apply ● Higher Order Thinking Skills

Test yourself

## First Multiple choice questions

**Choose the correct answer from those given :**

**( 1 )** In all the following relations , y is a function in $X$ except ...............

(a) $y = 3X + 1$ (b) $y = X^2 - 4$ (c) $X = y^2 - 2$ (d) $y = \sin X$

**( 2 )** In all the following relations , y is a function in $X$ except the relation ...............

(a) $y = \cos X$ (b) $y = 2$ (c) $y = X^2 - 1$ (d) $y^2 = X^2 + 1$

**( 3 )** If $f\left(\sqrt{X}\right) = X^2$ , then $f(2) =$ ...............

(a) $\sqrt{2}$ (b) 2 (c) 4 (d) 16

**( 4 )** If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = (a + 1)X + b - 2$ and $f(X)$ relates each real number to itself then (a , b) = ...............

(a) (1 , 3) (b) (0 , 3) (c) (0 , 2) (d) (0 , 0)

**( 5 )** The domain of the function $f : f(X) = 5$ is ...............

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\{5\}$ (d) $\{0 , 5\}$

**( 6 )** The domain of the function $f : f(X) = \frac{2X + 1}{X - 2}$ is ...............

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-\frac{1}{2}\}$ (c) $\mathbb{R} - \{-\frac{1}{2} , 2\}$ (d) $\mathbb{R} - \{2\}$

**( 7 )** The domain of the function $f : f(X) = \frac{X^2 + 1}{X^2 + 4X}$ is ...............

(a) $\mathbb{R} - \{1 , -1\}$ (b) $\mathbb{R} - \{0 , -4\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{0 , 4\}$

**(8)** The domain of the function $f$ where $f(x) = \frac{5 + 2x}{x^2 + x + 1}$ is ..............

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{5\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R} - \{-2, -5\}$

**(9)** The domain of the function $f$ where $f(x) = \frac{7}{x^3 - x}$ is ..............

(a) $\mathbb{R} - \{3\}$ (b) $\mathbb{R} - \{7\}$ (c) $\mathbb{R} - \{0, 1\}$ (d) $\mathbb{R} - \{0, 1, -1\}$

**(10)** The domain of the function $f$ where $f : \mathbb{R}^+ \longrightarrow \mathbb{R}$, $f(x) = \frac{x - 1}{4x}$ is ..............

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R}^+$ (d) $\mathbb{R} - \{1\}$

**(11)** If the domain of the function $f : f(x) = \frac{2}{x^2 - 6x + k}$ is $\mathbb{R} - \{3\}$, then k = ..............

(a) 3 (b) 9 (c) $\pm 9$ (d) 18

**(12)** The domain of the function $f : f(x) = \sqrt{x - 3}$ is ..............

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $[3, \infty[$ (d) $]-\infty, 3[$

**(13)** The domain of the function $f$ where $f(x) = \sqrt{4 - x}$ is ..............

(a) $[4, \infty[$ (b) $]-\infty, 4[$ (c) $]4, \infty[$ (d) $]-\infty, 4]$

**(14)** The domain of the function $f : f(x) = \sqrt[3]{x - 5}$ is ..............

(a) $[5, \infty[$ (b) $]-\infty, 5[$ (c) $\mathbb{R}$ (d) $\mathbb{R}^+$

**(15)** The domain of the function $f : f(x) = \sqrt[3]{9 - x^2}$ is ..............

(a) $]-3, 3[$ (b) $\mathbb{R}$ (c) $\mathbb{R} - ]-3, 3[$ (d) $[-3, 3]$

**(16)** The domain of the function $f : f(x) = \frac{5}{\sqrt{x - 4}}$ is ..............

(a) $[4, \infty[$ (b) $]4, \infty[$ (c) $]-\infty, 4]$ (d) $]-\infty, 4[$

**(17)** The domain of the function $f : f(x) = \frac{1}{\sqrt{9 - x^2}}$ is ..............

(a) $\mathbb{R}$ (b) $\mathbb{R} - [-3, 3]$ (c) $\mathbb{R} - \{-3, 3\}$ (d) $]-3, 3[$

**(18)** The domain of the function $f$ where $f(x) = \sqrt[4]{x^2 + 4}$ is ..............

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{4\}$ (c) $\mathbb{R} - \{0\}$ (d) $\mathbb{R} - \{-2, 2\}$

**(19)** The domain of the function $f$ where $f(x) = \frac{1}{\sqrt[3]{x^2 - 5x - 6}}$ is ..............

(a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R} - \{6\}$ (c) $\mathbb{R} - \{1, -6\}$ (d) $\mathbb{R} - \{-1, 6\}$

(20) If A is the area of a circle and $X$ is its radius length and the area is given as a function of $X$ where $A = \pi X^2$, then its domain = ..............

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{Z}^+$ (d) $\mathbb{R}^+$

(21) If the domain of the function $f : f(X) = \frac{1}{\sqrt{X - a}}$ is $]-3, \infty[$, then a = ..............

(a) 3 (b) $-3$ (c) $\pm 3$ (d) $\sqrt{3}$

(22) If the domain of the function $f : f(X) = \frac{1}{\sqrt{X^2 + a}}$ is $\mathbb{R} - [-5, 5]$, then a = ..............

(a) 5 (b) $-25$ (c) $-5$ (d) 25

(23) If the domain of the function $f : f(X) = \frac{1}{\sqrt{X^2 + a}}$ is $\mathbb{R}$, then a can not be equal ..............

(a) 5 (b) $\sqrt{4}$ (c) zero. (d) 9

(24) If $f(X) = \begin{cases} -4X + 3 , & X < 3 \\ -X^3 , & 3 \le X \le 8 \\ 3X^2 + 1 , & X > 8 \end{cases}$, then $f(10)$ = ..............

(a) $-37$ (b) $-1000$ (c) 301 (d) 43

(25) The domain of the function $f$ where $f(X) = \begin{cases} -2 , & X < 2 \\ 3 , & X > 2 \end{cases}$ is ..............

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\mathbb{R} - \{-2\}$ (d) $\mathbb{R} - \{2\}$

(26) The domain of the function $f$ where $f(X) = \begin{cases} X , & 0 \le X \le 1 \\ 2 - X , & 1 < X \le 2 \end{cases}$ is ..............

(a) $\mathbb{R} - \{1\}$ (b) $[0, 2]$ (c) $\mathbb{R} - \{0, 2\}$ (d) $]0, 2[$

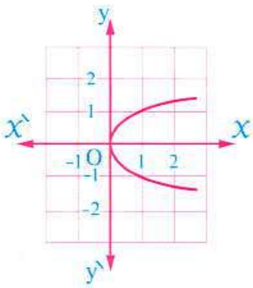
(27) The domain of the function $f : f(X) = \begin{cases} X^2 + 2 , & -2 \le X < 0 \\ X - 1 , & X > 0 \end{cases}$ is ..............

(a) $\mathbb{R}$ (b) $[-2, 0[$ (c) $[-2, \infty[ - \{0\}$ (d) $[-2, \infty[$

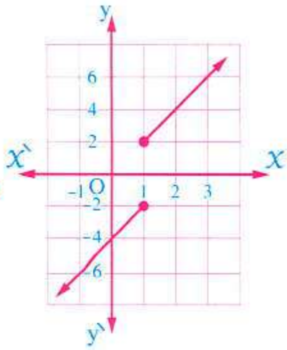
(28) The range of the function $f : f(X) = \begin{cases} 0 , & X \le 0 \\ 1 , & X > 0 \end{cases}$ is ..............

(a) $\{1\}$ (b) $\{0\}$ (c) $\mathbb{R}$ (d) $\{0, 1\}$

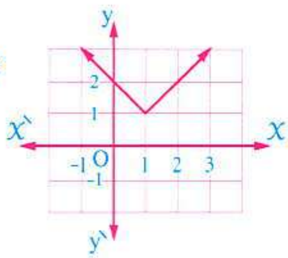
**(29)** The figure which represents y as a function in $x$ is ..............



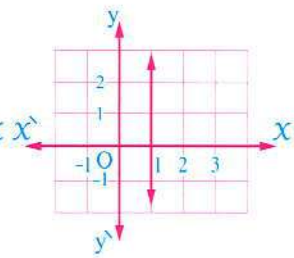

(a)




(b)

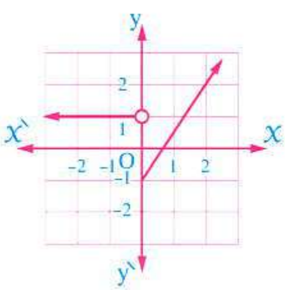



(c)

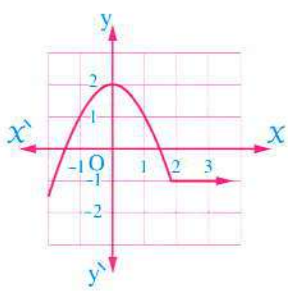



(d)

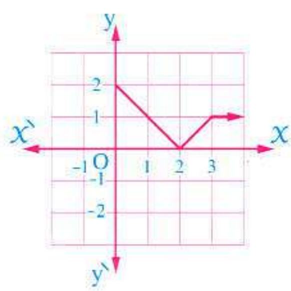
**(30)** Which of the following graphs does not represent a function ?



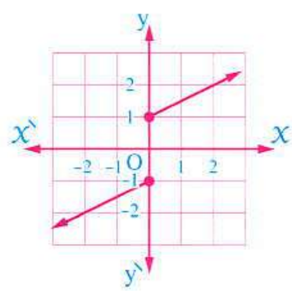

(a)




(b)




(c)

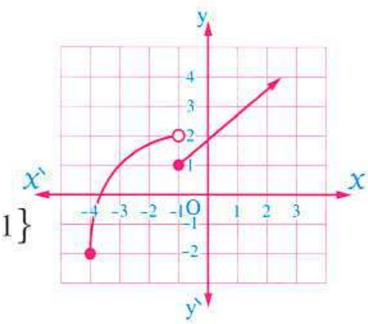



(d)

**(31)** The opposite figure represents the curve of function $f$ , then its domain is ..............

(a) $\mathbb{R} - \{-4 , -1\}$ (b) $]-4 , -1[$

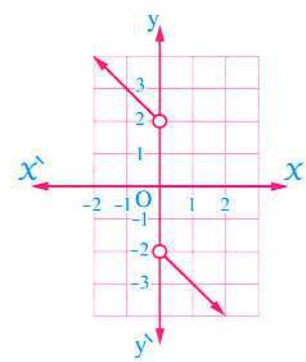
(c) $[-4 , \infty[$ (d) $[-4 , \infty[ - \{-1\}$



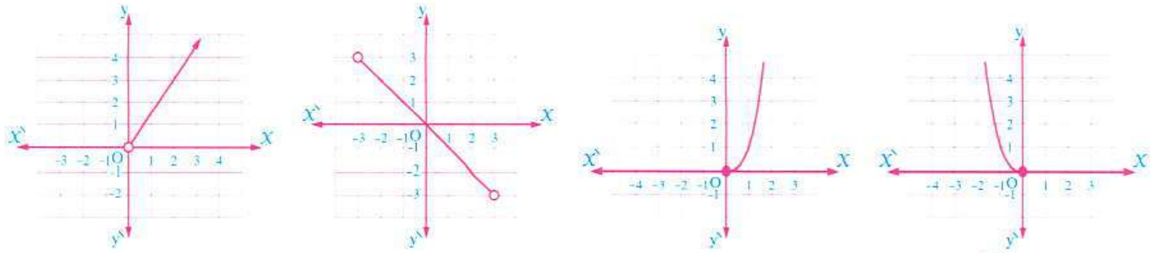

**(32)** The opposite figure represents function of $x$ , its domain is ..............

(a) $\mathbb{R}$ (b) $\mathbb{R} - ]-2 , 2[$

(c) $\mathbb{R} - [-2 , 2]$ (d) $\mathbb{R} - \{0\}$

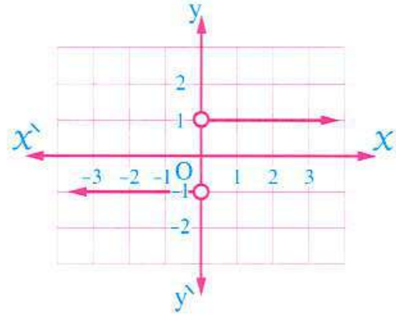


● **(33)** Which of the following figures represents the curve of a function in which its range $\neq$ its domain ?



(a) (b) (c) (d)

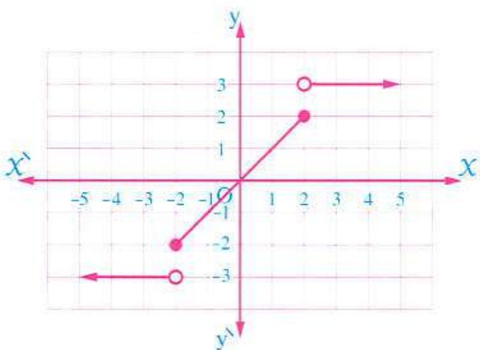
● **(34)** The range of the function shown in the opposite figure is ..............




(a) $\{1\}$ (b) $\{1 , -1\}$

(c) $\{-1\}$ (d) $\mathbb{R} - \{0\}$

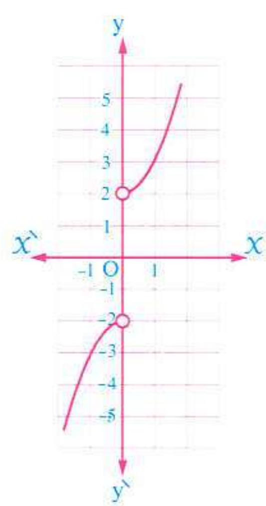
● **(35)** The opposite figure represents the function $f$, then its range is ..............




(a) $\mathbb{R}$ (b) $[-2 , 2]$

(c) $[-2 , 2] \cup \{-3 , 3\}$ (d) $]-3 , 3[$

● **(36) In the opposite figure :**




**First :** The range of the function is ..............

(a) $\mathbb{R} - \{0\}$ (b) $\mathbb{R} - [-2 , 2]$

(c) $\mathbb{R}$ (d) $[-2 , 2]$

**Second :** The function is increasing in ..............

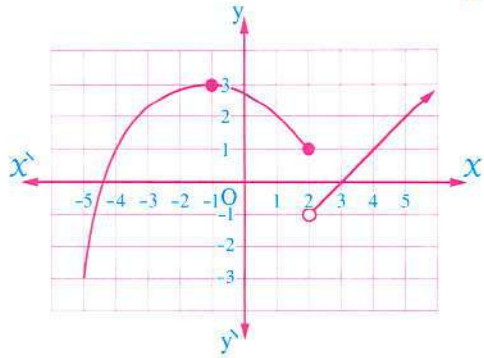
(a) $]-\infty , 0[$ only

(b) $]0 , \infty[$ only

(c) $]-\infty , 0[ , ]0 , \infty[$

(d) $\mathbb{R} - [-2 , 2]$

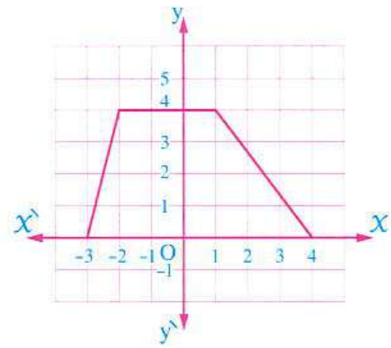
(37) The opposite figure represents the curve of the function $f$ which is increasing on ..............



(a) $]-\infty , -1[$ (b) $]-\infty , -1[ , ]2 , \infty[$

(c) $]2 , \infty[$ (d) $]-1 , 2[$

(38) The opposite figure represents the curve of the function $f$ which of the following statements is false ?



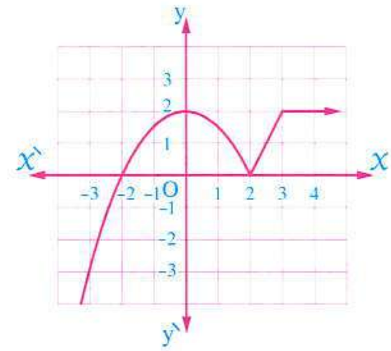
(a) $f$ is constant on $]-2 , 1[$

(b) $f$ is decreasing on $]1 , 4[$

(c) $f$ is increasing on $]-3 , -2[$

(d) $f$ is constant on $]-3 , 4[$

(39) In the opposite figure if the function decreases on $]0 , a[$ and constant on $]b , \infty[$ , then $a - b =$ ..............



(a) 5 (b) 1

(c) $-1$ (d) 3

## Second Essay questions

**1** **If $x$ and y are two real variables , then determine which of the following relations represents a function in $x$ :**

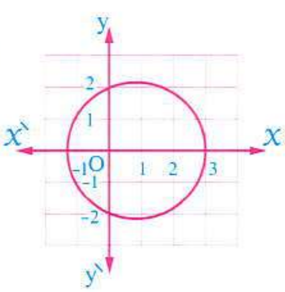
(1) $y = 2x + 5$ (2) $y^2 = x + 4$ (3) $y = \sqrt{x^2 + 4}$

(4) $(x - y)^2 = 5$ (5) $y^3 - x^2 = 2$ (6) $y = 2$

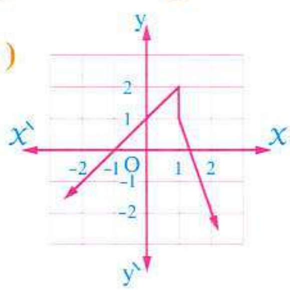
(7) $x = 3$ (8) $y = 3 \sin x$ (9) $xy + y = 2x + 2$

**2** **In each of the following graphs , show if y is a function in $x$ or not :**

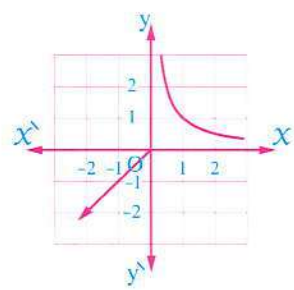
(1)



(2)



(3)



**3** Determine the domain of each of the real functions defined by the following rules :

(1) $f(x) = \dfrac{2x+3}{x^2-3x+2}$

(2) $f(x) = \dfrac{8}{x^2-6x+9}$

(3) $f(x) = \dfrac{x+3}{3x^2-x-2}$

(4) $f(x) = \dfrac{x+1}{x^3+1}$

**4** Determine the domain of each of the real functions defined by the following rules :

(1) $f(x) = \dfrac{4}{\sqrt[3]{2x-5}}$

(2) $f(x) = \dfrac{3}{\sqrt{x-3}}$

(3) $f(x) = \dfrac{2}{\sqrt{1-x}}$

(4) $f(x) = \sqrt{x^2-16}$

(5) $f(x) = \dfrac{5}{\sqrt{9-x^2}}$

(6) $f(x) = \sqrt{x^2+2x+5}$

(7) $f(x) = \dfrac{1}{\sqrt{x^2-4x+4}}$

(8) $f(x) = \dfrac{5}{\sqrt{x}-1}$

**5** Determine the domain of each of the real functions defined by the following rules :

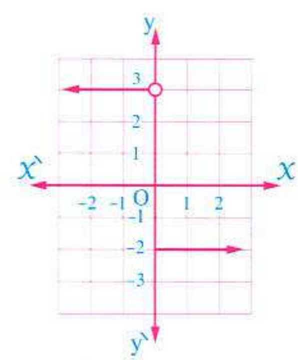
(1) $f(x) = \begin{cases} -3 & , \quad x < 3 \\ 5-x & , \quad x \geq 3 \end{cases}$

(2) $f(x) = \begin{cases} x^2-1 & , \quad x \leq 2 \\ -5 & , \quad 2 < x < 4 \end{cases}$

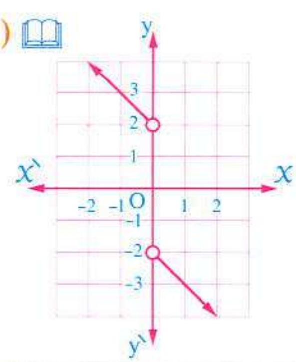
(3) $f(x) = \begin{cases} 3x & , \quad x \in [0, 2] \\ 6 & , \quad x \in ]2, 4[ \\ x+2 & , \quad x \in [4, 6] \end{cases}$

**6** Determine the domain and range, then discuss the monotony of each of the functions represented by the following graphs :

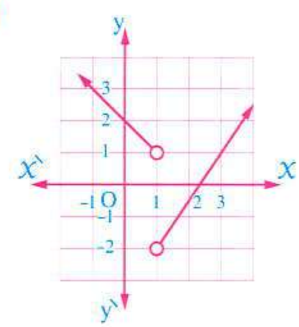
(1)



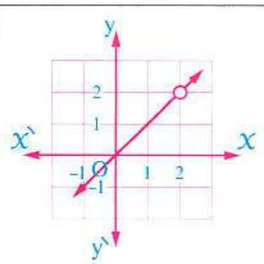
(2)



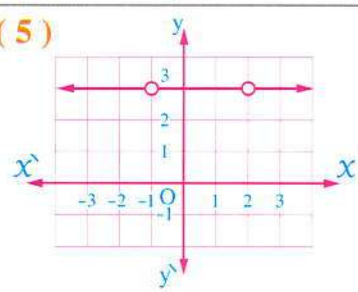
(3)



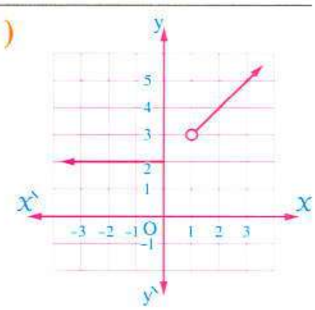
(4)



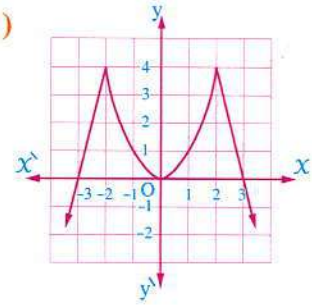
(5)



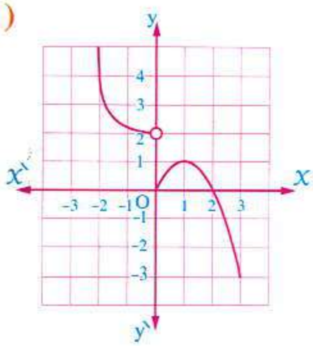
(6)



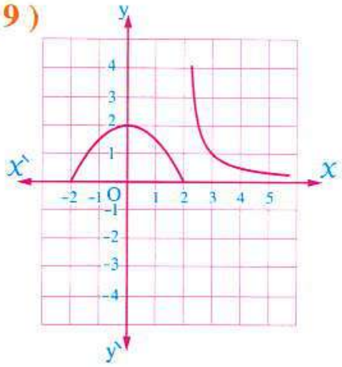
(7)



(8)



(9)



**Third** **Higher skills**

**Choose the correct answer from those given :**

(1) If the relation between the sum (y) of the interior angle measures of a polygon and the number of its sides ($X$) is $y = \pi (X - 2)$ , then the domain of this function is ...............

(a) $\mathbb{R}^+$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{Z}^+$ (d) $\mathbb{Z}^+ - \{1 , 2\}$

(2) The domain of the function $f : f(X) = \frac{X}{\sqrt[3]{X} - 2}$ is ...............

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{0 , 2\}$ (d) $\mathbb{R} - \{8\}$

(3) The domain of the function $f : f(X) = \frac{X}{\sqrt{3X} - X}$ is ...............

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[ - \{1\}$ (d) $]0 , \infty[ - \{3\}$

(4) The domain of the function $f : f(X) = \frac{5}{\sqrt{X-1} - 3}$ is ...............

(a) $[1 , \infty[$ (b) $[1 , \infty[ - \{3\}$ (c) $[1 , \infty[ - \{10\}$ (d) $[-3 , \infty[$

(5) The domain of the function $f : f(X) = \frac{7}{\sqrt{X-1} + 3}$ is ...............

(a) $[1 , \infty[$ (b) $[1 , \infty[ - \{-3\}$ (c) $[1 , \infty[ - \{10\}$ (d) $[3 , \infty[$

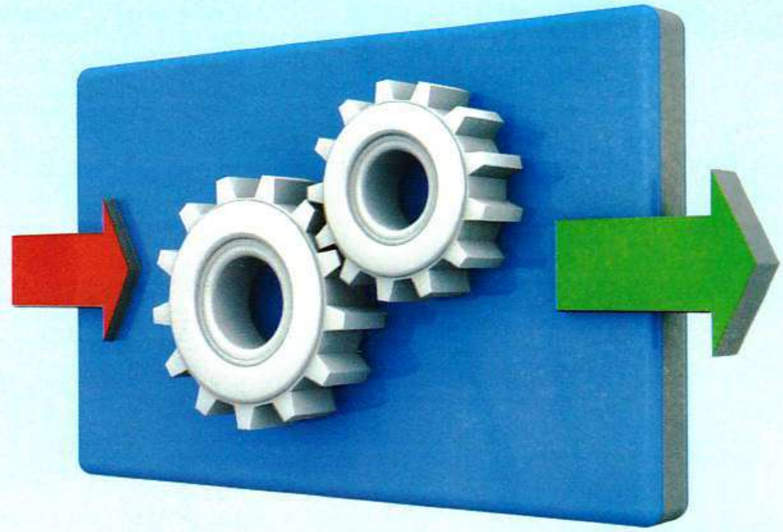
(6) The domain of the function $f : f(X) = \sqrt{6X - 9 - X^2}$ is ...............

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $[3 , \infty[$ (d) $\{3\}$

Exercise

# 2

## Operations on functions – Composition of functions



From the school book  Understand  Apply  Higher Order Thinking Skills

Test yourself

## First Multiple choice questions

**Choose the correct answer from those given :**

**( 1 )** If $f(X) = \sqrt[3]{X - 1}$ , $g(X) = \sqrt{X - 1}$ , then the domain of $(f - g)$ is ..............

(a) $[-1 , \infty[$ (b) $[1 , \infty[$ (c) $\mathbb{R}$ (d) $]-\infty , 1]$

**( 2 )** The domain of the function $f : f(X) = \sqrt{X - 1} + \sqrt{X + 2}$ is ..............

(a) $[-2 , \infty[$ (b) $[1 , \infty[$ (c) $]1 , \infty[$ (d) $]-2 , \infty[$

**( 3 )** The domain of the function $f : f(X) = \sqrt{X + 2} - \sqrt{3 - X}$ is ..............

(a) $\mathbb{R}$ (b) $\mathbb{R} - [-2 , 3]$ (c) $[-2 , 3]$ (d) $]-2 , 3[$

**( 4 )** If $f$ is a function where $f = \{(1 , 2) , (2 , 4) , (3 , 6) , (4 , 8)\}$ and g is a function where $g = \{(1 , 3) , (3 , 5) , (5 , 7)\}$ , then $f + g =$ ..............

(a) $\{(1 , 2) , (2 , 4) , (3 , 6) , (4 , 8) , (1 , 3) , (3 , 5) , (5 , 7)\}$

(b) $\{(1 , 2) , (2 , 4) , (3 , 6) , (4 , 8) , (5 , 7)\}$

(c) $\{(1 , 5) , (3 , 11)\}$

(d) $\{(1 , 5) , (2 , 4) , (3 , 11) , (4 , 8) , (5 , 7)\}$

**( 5 )** If $f , g : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 3X + 1$ , and $(f + g)(X) = X^3 + 2X - 1$ , then $g(-1) =$ ..............

(a) $-2$ (b) $-1$ (c) zero (d) 2

**( 6 )** If $f : \mathbb{R}^+ \longrightarrow \mathbb{R}$ where $f(x) = x^2 - 7$ , $g : [-3 , 2] \longrightarrow \mathbb{R}$ where $g(x) = x^2 + 3$ , then $(f - g)(-5) =$ ...............

(a) $-5$ (b) $-10$ (c) $-20$ (d) undefined.

**( 7 )** The domain of the function (f . g) where $f(x) = x^2 - 4$ , $g(x) = x + 1$ is ...............

(a) $\mathbb{R} - \{2 , -2 , -1\}$ (b) $\{2 , -2 , -1\}$

(c) $\mathbb{R}$ (d) $\mathbb{R} - \{4 , 1\}$

**( 8 )** If $f(x) = \frac{1}{x}$ , $g(x) = \sqrt{x}$ , then the domain of $(f \times g) =$ ...............

(a) $\mathbb{R} - \{0\}$ (b) $\mathbb{R}$ (c) $\mathbb{R}^+$ (d) $[0 , \infty[$

**( 9 )** The domain of the function $f : f(x) = \sqrt{x - 4}\ \sqrt[3]{x - 2}$ is ...............

(a) $]4 , \infty[$ (b) $[2 , \infty[$ (c) $\mathbb{R}$ (d) $[4 , \infty[$

**(10)** If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = x^2 - x$ , $g : \mathbb{R} \longrightarrow \mathbb{R}$ where $g(x) = 3x - 2$ , then $(f \times g - 6g)(2) =$ ...............

(a) 24 (b) 40 (c) 16 (d) $-16$

**(11)** If $f(x) = 2x + 1$ , $g(x) = -x - 2$ , $h(x) = f(x) + g(x)$ , $k(x) = f(x) - g(x)$ , then $h(2) \times k(1) =$ ...............

(a) 3 (b) 1 (c) 6 (d) zero

**(12)** If $f : \mathbb{R}^+ \longrightarrow \mathbb{R}$ where $f(x) = x - 5$ , $g : [-1 , 5] \longrightarrow \mathbb{R}$ where $g(x) = x - 2$ , then $\left(\frac{f}{g}\right)(1)$ ...............

(a) 1 (b) 2 (c) 4 (d) 8

**(13)** If $f$ , g are two real functions where $f(x) = \frac{x - 2}{x^2 - 3x + 2}$ , $g(x) = x - 3$ , then $\left(\frac{f}{g}\right)(3) =$ ...............

(a) $\frac{1}{2}$ (b) 1 (c) zero (d) undefined.

**(14)** If $f$ , g are polynomial functions and the domain of the function $\left(\frac{f}{g}\right)$ is $\mathbb{R} - \{3\}$ and the domain of the function $\left(\frac{g}{f}\right)$ is $\mathbb{R} - \{2\}$ , then the domain of $(f . g)$ is ...............

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R} - \{2 , 3\}$

**(15)** The domain of the function $f : f(x) = \frac{x - 5}{\sqrt{2x - 3}}$ is ...............

(a) $\mathbb{R} - \{5\}$ (b) $]\frac{3}{2} , \infty[$ (c) $[\frac{3}{2} , \infty[ - \{5\}$ (d) $\mathbb{R} - \{\frac{3}{2}\}$

**(16)** The domain of the function $f : f(x) = \frac{\sqrt{x-2}}{x-3}$ is ...............

(a) $\mathbb{R}$ (b) $\{3\}$ (c) $[2, \infty[$ (d) $[2, \infty[ - \{3\}$

**(17)** If $f : \mathbb{R}^+ \longrightarrow \mathbb{R}$ where $f(x) = x - 5$, $g : [-1, 5] \longrightarrow \mathbb{R}$ where $g(x) = x - 2$, then the domain of $\left(\frac{f}{g}\right)$ is ...............

(a) $]0, 5]$ (b) $]0, 5] - \{2\}$ (c) $[-1, \infty[$ (d) $\mathbb{R}$

**(18)** If $f_1 : [-2, 3] \longrightarrow \mathbb{R}$, where $f_1(x) = x$, $f_2 : [0, 4[ \longrightarrow \mathbb{R}$, where $f_2(x) = x^2$, then the domain of $\left(\frac{f_1}{f_2}\right) =$ ...............

(a) $[-2, 4[$ (b) $[-2, 4[ - \{0\}$ (c) $]0, 3]$ (d) $[0, 3]$

**(19)** The domain of the function $f$ where $f(x) = \frac{x+1}{\sqrt[3]{x-1}}$ is ...............

(a) $\mathbb{R} - \{1\}$ (b) $\mathbb{R} - \{-1\}$ (c) $[-1, \infty[$ (d) $\mathbb{R}$

**(20)** If $f(x) = \sqrt[3]{x-3}$, $g(x) = \sqrt{3-x}$, then the domain of $\left(\frac{f}{g}\right)$ is ...............

(a) $]3, \infty[$ (b) $[3, \infty[$ (c) $]-\infty, 3[$ (d) $]-\infty, 3]$

**(21)** The domain of the function $f : f(x) = \frac{\sqrt{x-4}}{(x-3)(x-5)}$ is ...............

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3, 5\}$ (c) $[4, \infty[$ (d) $[4, \infty[ - \{5\}$

**(22)** The domain of the function $f$ where $f(x) = \frac{\sqrt{x-3}}{\sqrt{5-x}}$ is ...............

(a) $]3, 5]$ (b) $\mathbb{R} - \{3, 5\}$ (c) $[3, 5[$ (d) $\{-3, -5\}$

**(23)** The domain of the function $f : f(x) \sqrt{\frac{3-x}{5-x}}$ is ...............

(a) $]3, 5]$ (b) $\mathbb{R} - ]3, 5]$ (c) $[3, 5]$ (d) $\mathbb{R} - [3, 5[$

**(24)** If $f(x) = \sqrt{x+5}$, $g(x) = x^2$, then $(f \circ g)(2) =$ ...............

(a) 7 (b) 3 (c) 4 (d) 9

**(25)** If $f(1) = 4$, $g(4) = 7$, then $(g \circ f)(1) =$ ...............

(a) 1 (b) 4 (c) 7 (d) 11

**(26)** If $f : [-2, 3] \longrightarrow \mathbb{R}$ where $f(x) = x + 1$, $g : [0, 2] \longrightarrow \mathbb{R}$ where $g(x) = x^2$, then $(f \circ g)(2) =$ ...............

(a) 4 (b) $-1$ (c) 5 (d) undefined.

**(27)** If $f(X) = 5X + 4$ , $g(X) = 2 - X$ , then $(f \circ g)(-2) + (g \circ f)(5) =$ ..............

(a) 24 (b) $-27$ (c) 49 (d) $-3$

**(28)** If $f(X) = \sqrt[3]{X}$ , $g(X) = X^3$ , then $(f \circ g)(X) =$ ..............

(a) $X$ (b) $X^3$ (c) $\sqrt[3]{X}$ (d) $|X|$

**(29)** If $f(X) = X - 3$ , $g(X) = X^2$ , then $(f \circ g)(X) =$ ..............

(a) $(X - 3)^2$ (b) $X^2 - 3$ (c) $X^2 + 3$ (d) $\sqrt{X - 3}$

**(30)** If $f(X) = \frac{3X + 2}{2X - 3}$ , then $(f \circ f)(X) =$ ..............

(a) $X$ (b) $-X$ (c) $f(X)$ (d) $-f(X)$

**(31)** If $f(X) = 2X + 5$ and $g(X) = X^2$ , then $f\left(g(X)\right) =$ ..............

(a) $(2X + 5)^2$ (b) $2X^2 + 5$ (c) $(2X^2 + 5)^2$ (d) $2X^2 + 10$

**(32)** If $f(X) = 3X + 2$ , $g(X) = 2X + k$ and $(f \circ g)(X) = (g \circ f)(X)$ , then $k =$ ..............

(a) 1 (b) 2 (c) 3 (d) 4

**(33)** If $f(X) = X + 1$ , $g(X) = X^2 - 1$ , then the solution set of the equation $g\left(f(X)\right) =$ zero is ..............

(a) $\{0 , 2\}$ (b) $\{1 , -1\}$ (c) $\{0 , -2\}$ (d) $\{2 , -2\}$

**(34)** If $f(X) = aX + 3$ , $g(X) = 2X + a$ and $(f \circ g)(2) = 15$ , then $a \in$ ..............

(a) $\{2 , 6\}$ (b) $\{-2 , -6\}$ (c) $\{-6 , 2\}$ (d) $\{-2 , 6\}$

**(35)** If $f(X) = 2X$ , $g(X) = \frac{1}{2X}$ , $f\left(g(X)\right) = 6$ , then $X =$ ..............

(a) $\frac{1}{6}$ (b) 6 (c) 4 (d) $\frac{1}{12}$

**(36)** If $f(X) = \frac{1}{X}$ , $g(X) = X^2 - 1$ , then the domain of $(f \circ g) =$ ..............

(a) $\{0 , 1 , -1\}$ (b) $\mathbb{R} - \{0 , 1 , -1\}$ (c) $\mathbb{R} - \{1 , -1\}$ (d) $[-1 , 1]$

**(37)** If $f(X) = \sqrt{X}$ , $g(X) = X^2$ , then the domain of $(g \circ f) =$ ..............

(a) $]-\infty , 0]$ (b) $[0 , \infty[$ (c) $\mathbb{R}^+$ (d) $\mathbb{R}$

**(38)** If $f(X) = \sqrt{X}$ , $g(X) = X^2$ , then the domain of $(f \circ g) =$ ..............

(a) $]-\infty , 0]$ (b) $[0 , \infty[$ (c) $\mathbb{R}^+$ (d) $\mathbb{R}$

**(39)** The domain of the function $(f \circ g)$ where :

$f(X) = \sqrt{X - 2}$ , $g(X) = \sqrt{6 - X}$ is ..............

(a) $\mathbb{R}$ (b) $[2 , \infty[$ (c) $]-\infty , 2]$ (d) $]-\infty , 2[$

**(40)** If the relation between $x$ , $f(x)$ , $g(x)$ is as given in the opposite table , then $g(f(6)) = \ldots\ldots$

| $x$ | $f(x)$ | $x$ | $g(x)$ |
|---|---|---|---|
| −3 | 4 | −4 | 3 |
| −2 | 3 | −1 | 1 |
| 3 | 1 | 1 | −4 |
| 6 | −1 | 4 | −7 |

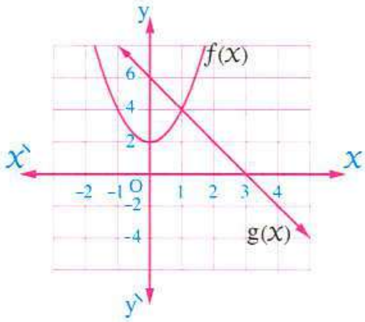
(a) 3 (b) 1
(c) −4 (d) −7

**(41)** If the opposite table shows two functions $f$ and g , then $(f \circ g)(1) = \ldots\ldots$

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f(x)$ | 3 | 1 | 4 | 2 |
| $g(x)$ | 4 | 3 | 2 | 1 |

(a) 1 (b) 2
(c) 3 (d) 4

**(42)** The opposite figure shows the curves of two functions $f$ and g , then $(g \circ f)(1) = \ldots\ldots$




(a) 5 (b) −2
(c) −4 (d) 6

## Second Essay questions

**1** **Let $f$ be a function such that : $f(x) = x^2 - x - 12$ and its domain is $[-4, 8]$ , g is another function such that : $g(x) = x - 4$ and its domain is $[-7, 4]$ , find each of the following functions and determine the domain of each of them :**

(1) $(f + g)(x)$ (2) $(f - g)(x)$ (3) $(f \cdot g)(x)$
(4) $\left(\frac{f}{g}\right)(x)$ (5) $\left(\frac{g}{f}\right)(x)$

---

**2** **If $f$ , g are two real functions where $f(x) = x^2 - 4$ , $g(x) = \sqrt{x - 1}$ , find :**

(1) The domain of each of the following functions : $(f + g)$ , $(f \cdot g)$ , $\left(\frac{f}{g}\right)$ , $\left(\frac{g}{f}\right)$

(2) The numerical value – if possible – for each of : $(f + g)(5)$ , $(f \cdot g)(2)$ , $\left(\frac{f}{g}\right)(3)$ , $\left(\frac{g}{f}\right)(-2)$

---

**3** **If $f(x) = x^2 - 4x$ , $g(x) = \sqrt{x + 2}$ , $h(x) = \sqrt{4 - x}$**

**First : Find the rule and the domain of each of the following functions :**

(1) $(f + g)$ (2) $(g - h)$ (3) $(f \cdot h)$ (4) $\left(\frac{h}{f}\right)$

**Second : Evaluate the numerical value – if possible – for each of :**

(1) $(g - h)(1)$ (2) $(f \cdot h)(5)$ (3) $\left(\frac{h}{f}\right)(3)$

**4** **Determine the domain of each of the real functions defined by the following rules :**

(1) $f(x) = \frac{1}{x} + \frac{1}{x+2}$

(2) $f(x) = \frac{x-1}{x^2-1} + \frac{1}{x+1}$

(3) $f(x) = \frac{x}{\sqrt{1-x}}$

(4) $f(x) = \frac{3x}{\sqrt{2x-1}}$

(5) $f(x) = \sqrt[3]{x-4} + \sqrt{x-2}$

(6) $f(x) = \frac{\sqrt{x-4}}{x-6}$

(7) $f(x) = \frac{\sqrt{x-3}}{x^2-4}$

(8) $f(x) = \frac{\sqrt{x-3}}{\sqrt{x-5}}$

---

**5** **If $f$ , g are two real functions where :**

$f(x) = \begin{cases} x-2 & , \quad x \geq 2 \\ -x+2 & , \quad x < 2 \end{cases}$ , **$g(x) = x$ , then find each of the following :**

(1) $(f \cdot g)(x)$

(2) $\left(\frac{f}{g}\right)(x)$

(3) $\left(\frac{g}{f}\right)(x)$

determining the domain of each function.

---

**6** **Determine the domain of each of the real functions defined by the following rules :**

(1) $f(x) = \sqrt{\frac{x-3}{x-5}}$

(2) $f(x) = \frac{5x+2}{\sqrt{x-2} - 1}$

(3) $f(x) = \frac{\sqrt{x+2}}{x^2-6x-7}$

(4) $f(x) = \frac{\frac{3}{x-1}}{\frac{1}{x-2}+4}$

---

**7** **If $f(x) = 3x+1$ , $g(x) = x^2-5$ , $k(x) = x^3$ , find :**

(1) $(f \circ g)(2)$

(2) $(g \circ f)(-3)$

(3) $(g \circ k)(1)$

(4) $(k \circ f)(-2)$

« −2 , 59 , −4 , −125 »

---

**8** **If $f(x) = \frac{1}{x}$ , $g(x) = x+3$ , find :**

(1) $(f \circ g)(x)$

(2) $(g \circ f)(x)$

and deduce the domain of each function.

---

**9** **If $f(x) = x^2+6$ , $g(x) = 3x$**

(1) Find : $(f \circ g)(3)$

(2) Determine the values of $x$ that make $(f \circ g)(x) = 42$

« 87 , ±2 »

**10** **If $f(x) = x^2 - 3$ , $g(x) = \sqrt{x-2}$ , find $(f \circ g)(x)$ in the simplest form , determining its domain , then find $(f \circ g)(3)$** « – 2 »

**11** **If $f(x) = \sqrt{x+1}$ , $g(x) = \frac{2}{x-3}$ , then find the domain of $(f \circ g)$**

**12** **If $f(x) = \sqrt{x-2}$ , $g(x) = \sqrt{4-x}$ , then find each of the following functions showing the domain :**

(1) $f \circ g$ (2) $g \circ f$

**13** **If $f(x) = \sqrt{x-1}$ , $g(x) = \sqrt{x^2-4}$ , then find each of the following functions :**

(1) $f \circ g$ (2) $g \circ f$

**14** **If $h(x) = \sqrt{x^3-4}$ , then find the two functions $f$ and g where :** $h(x) = (f \circ g)(x)$

**15** **If $f$ is a linear function and $(f \circ f)(x) = 16x + 15$, find :** $f(x)$

## Third Higher skills

**1** **Choose the correct answer from those given :**

(1) If $f(x) = \sqrt{x-1}$ , $g(x) = \sqrt{1-x}$ , then the domain of $(f + g)$ is ..............

(a) $[1 , \infty[$ (b) $]-\infty , 1]$ (c) $[-1 , \infty[$ (d) $\{1\}$

(2) If $f(x) = \begin{cases} -x+2 , & x > 0 \\ 3x , & x \leq 0 \end{cases}$ , $g(x) = \begin{cases} 2x-1 , & x \geq 1 \\ x+2 , & x < 1 \end{cases}$

, then $(f + g)(x) =$ .............. where $0 < x < 1$

(a) $4x + 2$ (b) $x + 1$ (c) $5x - 1$ (d) 4

(3) If $f(x) = \begin{cases} x^2+1 , & x < 0 \\ 3x+1 , & x \geq 0 \end{cases}$ , $g(x) = \begin{cases} 4x-2 , & x < 2 \\ x^2+2 , & x \geq 2 \end{cases}$

, then $(g \circ f)(-1) =$ ..............

(a) 37 (b) – 17 (c) 10 (d) 6

(4) If $f(x) = x^2$ , $g(x) = 2x + 3$ , then the solution set of the equation $f(g(x)) = f(x)$ is ..............

(a) $\{1 , 3\}$ (b) $\{-3 , 1\}$ (c) $\{-1 , -3\}$ (d) $\{-1 , 3\}$

( 5 ) If $f(x) = 2x + 1$ and $(f \circ g)(x) = 3x + 2$ , then $g(x) =$ ..............

(a) $x + 1$ (b) $5x + 3$ (c) $\frac{3}{2}x + \frac{1}{2}$ (d) $2x + 3$

( 6 ) If $f : \mathbb{R} \longrightarrow \mathbb{R}$ and $f(x) = 2x - 3$ , $(g \circ f)(x) = x + 2$ , then $g(x) =$ ..............

(a) $2x^2 + x - 6$ (b) $\frac{x+2}{2x-3}$ (c) $\frac{x+3}{2}$ (d) $\frac{x+7}{2}$

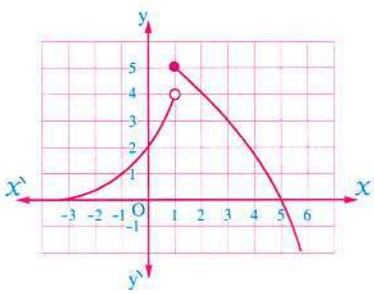
( 7 ) If $f(x) = x + 2$ , $g(x) = \frac{x - k}{3x - 1}$ and $(g \circ f)(0) = (f \circ g)(1)$ , then $k =$ ..............

(a) 7 (b) 5 (c) 1 (d) $\frac{7}{2}$

( 8 ) The opposite figure

represents the curve $y = f(x)$

, then $(f \circ f \circ f)(1) =$ ..............




(a) – 5 (b) – 1

(c) 1 (d) 2

( 9 ) If $f : \mathbb{N} \longrightarrow \mathbb{N}$ where $f(x) = 2x$ , $h : \mathbb{N} \longrightarrow \mathbb{N}$ where

$$h(x) = \begin{cases} \frac{x}{2} & , x \text{ is even} \\ \frac{x+1}{2} & , x \text{ is odd} \end{cases}$$

, then $(f \circ h)(3) - (f \circ h)(8) =$ ..............

(a) 4 (b) 8 (c) – 4 (d) – 5

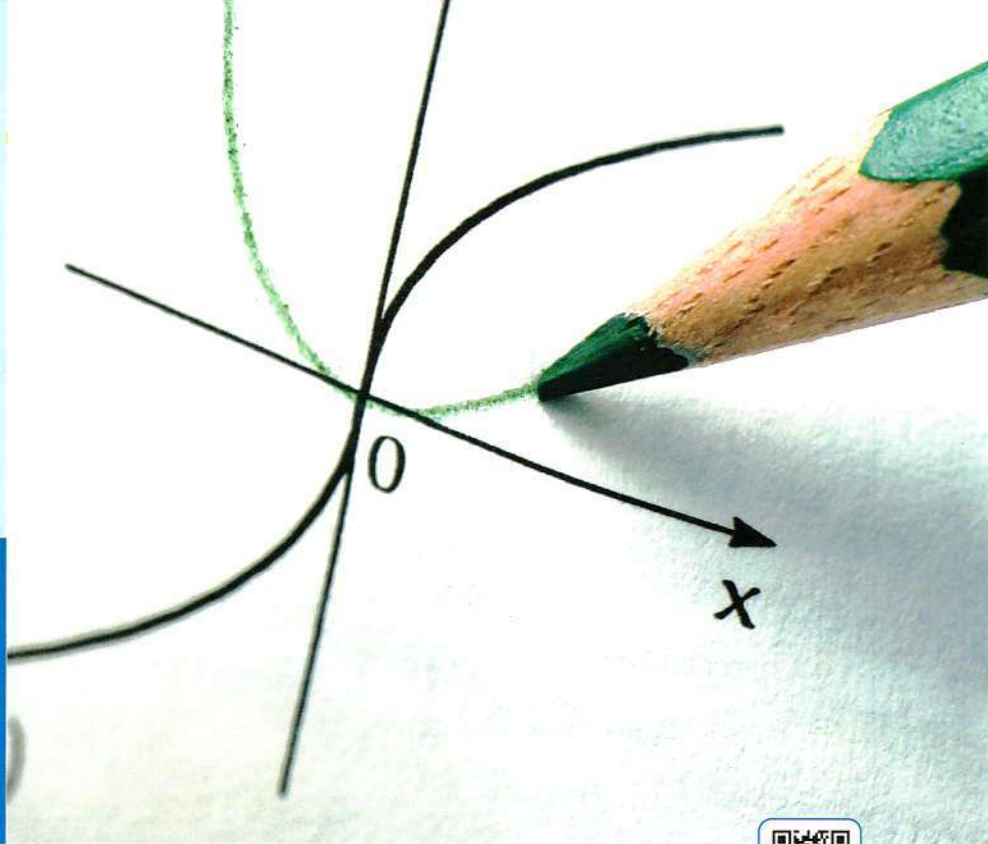
**2** **If $f : \mathbb{R} - \{0\} \longrightarrow \mathbb{R}$ and $f(x) = \frac{1}{x}$ , find :**

( 1 ) $(f \circ f)(x)$ ( 2 ) $(f \circ f \circ f)(x)$ ( 3 ) $(f \circ f \circ f \circ$ .... to n times$)(x)$

Exercise

# 3

## Some properties of functions (even and odd functions / one - to - one functions)




From the school book ● Understand ● Apply ● Higher Order Thinking Skills

Test yourself

## First Multiple choice questions

**Choose the correct answer from those given :**

( 1 ) The even function from the functions that are defined by the following rules is ...............

(a) $f(X) = X^3$ (b) $f(X) = \sin X$

(c) $f(X) = X \cos X$ (d) $f(X) = X \sin X$

( 2 ) The odd function from the functions that are defined by the following rules is ...............

(a) $f(X) = X^2 \sin X$ (b) $f(X) = \tan^2 X$

(c) $f(X) = \cos X$ (d) $f(X) = 1$

( 3 ) The type of the function $f : f(X) = \frac{\sin X}{X}$ is ...............

(a) even. (b) odd.

(c) neither even nor odd. (d) one - to - one.

( 4 ) The function $f : f(X) = X \cos X$ is ...............

(a) even. (b) odd.

(c) neither even nor odd. (d) one - to - one.

( 5 ) Each of the following is a rule of even function except ...............

(a) $f(X) = \sin X$ (b) $f(X) = \cos X$ (c) $f(X) = X^2 - 1$ (d) $f(X) = 1$

**( 6 )** Which of the following rules does not represent an even function ?

(a) $y = \frac{1}{x^2}$ (b) $y = \sec x$

(c) $y = x^2 + \sin x$ (d) $y = 3x^4 - 2x^2 + 27$

**( 7 )** If $f(x) = \frac{1}{\sin x}$ , then ...............

(a) $f(x) = \frac{1}{f(x)}$ (b) $f(x) = -f(-x)$

(c) $f(x) = f(-x)$ (d) $f(-x) = f\left(\frac{1}{x}\right)$

**( 8 )** If $f$ is an odd function , $f(1) = 2$ , then which of the following points lies on the curve of $f$ ?

(a) $(-1 , 2)$ (b) $(-1 , -2)$ (c) $(1 , -2)$ (d) $(-1 , 0)$

**( 9 )** If $f$ is an odd function , $a \in$ the domain of $f$ , then $f(a) + f(-a) =$ ...............

(a) zero. (b) $2f(a)$ (c) 2 a (d) $f(a)$

**(10)** If $f$ is an odd function , then $f(a) - f(-a) =$ ...............

(a) zero. (b) $f(a)$ (c) $2f(a)$ (d) $f(2a)$

**(11)** If $f$ is an even function , then $f(a) - f(-a) =$ ...............

(a) zero. (b) $f(a)$ (c) $2f(a)$ (d) $f(2a)$

**(12)** If $f$ is an even function , $2 \in$ the domain of $f$ , then $f(2) + f(-2) =$ ...............

(a) zero. (b) 4 (c) 2 (d) $2f(2)$

**(13)** The function $f : f(x) = (x + \sin x)$ (...............) is even.

(a) $x^3 + \tan x$ (b) $1 + \sin x$ (c) $4 - x^2$ (d) $x - \cos x$

**(14)** If $f$ is an even function and the curve of the function passes through point $(-3 , 2m + 1)$ , and $f(3) = 5$ , then m = ...............

(a) $-1$ (b) zero (c) 1 (d) 2

**(15)** 📖 If the function $f$ is an even over $[a , b]$ , then b = ...............

(a) a (b) $-a$ (c) 2 a (d) $a^3$

**(16)** If $f$ is an even function and $f(5) = 1$ , $f(-5) = 3 - k$ , then k = ...............

(a) 1 (b) 5 (c) 3 (d) 2

**(17)** If $f$ is a function where $f : ]-5 , 5] \longrightarrow \mathbb{R}$ , $f(x) = x^2$ , then the function $f(x)$ is ...............

(a) even. (b) odd.

(c) one - to - one. (d) neither odd nor even.

**(18)** If the domain of an even function $f$ is $]-5 , 5[$ , then the solution set of the equation $f(x) = f(x - 2)$ is ...............

(a) $\{2\}$ (b) $\{1\}$ (c) $\{0\}$ (d) $\{-1\}$

**(19)** If $f(X) = aX^3 + bX + c$ is an odd function , then c = ...............

(a) 2 (b) 1 (c) zero. (d) – 1

**(20)** If $f : f(X) = X^2 + aX + 9$ is an even function , then a = ...............

(a) 6 (b) 3 (c) zero. (d) – 6

**(21)** If $f(X) = X^3 - X$ , then $|f(X) + f(-X)|$ = ...............

(a) zero. (b) 1 (c) 2 (d) 4

**(22)** If $f$ is an even function and $f(X) + X^2 f(-X) = 3$ , then $f(1)$ = ...............

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{3}{2}$ (d) 2

**(23)** If $f$ is an odd function then : $\frac{2f(X) + 8f(-X)}{3f(X)}$ = ............... where $f(X) \neq$ zero

(a) zero (b) – 2 (c) – 4 (d) 4

**(24)** If $f$ is an odd function and $X f(X) + X^3 f(-X) = 2$ , then $f(2)$ = ...............

(a) – 2 (b) 2 (c) $\frac{-1}{3}$ (d) $\frac{1}{10}$

**(25)** If $f : f(X) = aX^3 + b$ is an odd function and the curve of the function passes through the point (2 , 8) , then $a + b^2$ = ...............

(a) zero (b) – 1 (c) 1 (d) 5

**(26)** The one - to - one function from those defined by the following rules is ...............

(a) $f(X) = 3 - X^2$ (b) $f(X) = X^3 + 3$

(c) $f(X) = \sin X \tan X$ (d) $f(X) = X^2 + X$

**(27)** The functions defined by the following rules are one - to - one except ...............

(a) $f(X) = X^3$ (b) $g(X) = 3X$ (c) $h(X) = \frac{1}{X}$ (d) $n(X) = X^2$

**(28)** All functions defined by the following rules are not one - to - one except ...............

(a) $f(X) = X^2$ (b) $f(X) = 5$ (c) $f(X) = X + 2$ (d) $f(X) = \sin X$

**(29)** If $f$ , g are two functions where $f(X) = X^3$ , $g(X) = X + 2$ , then $(g \circ f)$ is ............... function.

(a) a one - to - one (b) an odd (c) an even (d) a linear

**(30)** If the function is continuous and decreasing for all values of $X \in$ the domain of the function, then the function is ...............

(a) even. (b) odd. (c) one - to - one. (d) not one - to - one.

**(31)** If a function is increasing for all real values of $X \in$ the domain of the function , then the function is ...............

(a) one - to - one. (b) an even function.

(c) an odd function. (d) not one - to - one.

**(32)** The function $f : f(X) = X^3 + 5X$ is symmetric about ..............

(a) the $X$-axis. (b) the y-axis.

(c) the origin. (d) can not be determined.

**(33)** The function $f : f(X) = X^2 + X^4 + 1$ is symmetric about ..............

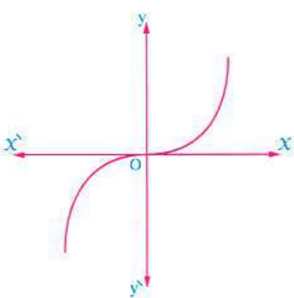
(a) the origin. (b) the $X$-axis.

(c) the y-axis. (d) it has neither symmetric point nor symmetric line.

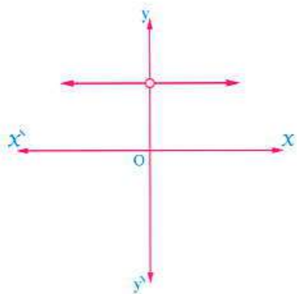
**(34)** The function $f : f(X) = \sin 3X$ is symmetric about the point ..............

(a) $(0 , 0)$ (b) $(3 , 0)$ (c) $(-3 , 0)$ (d) $(-3 , 3)$

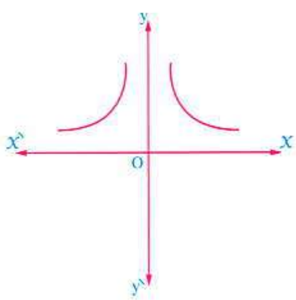
**(35)** Which of the following functions is not even ?



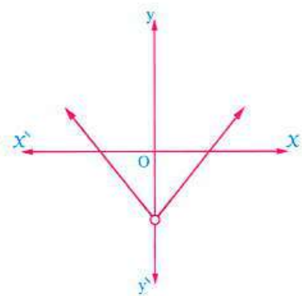
(a)



(b)

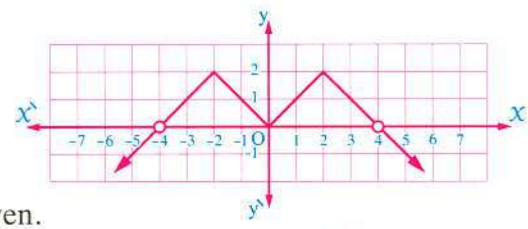


(c)



(d)

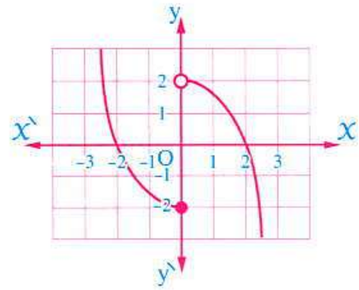
**(36)** The opposite figure represents the curve of the function $f$ , then $f$ is ..............



(a) one - to - one. (b) an even function.

(c) an odd function. (d) neither odd nor even.

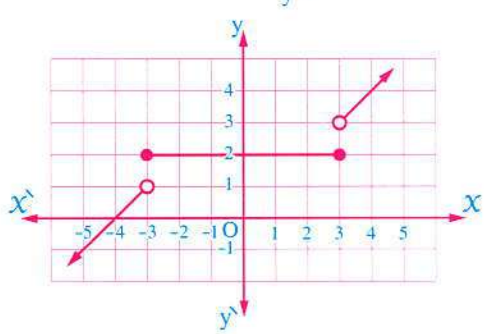
**(37)** The opposite figure represents the curve of the function $f$ , then $f$ is ..............



(a) one - to - one. (b) an even function.

(c) an odd function. (d) neither odd nor even.

**(38)** The opposite figure represents the curve of the function $f$ , then $f$ is ..............



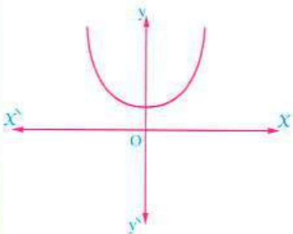
(a) one - to - one function.

(b) an even function.

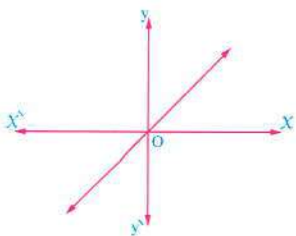
(c) an odd function.

(d) neither odd nor even.

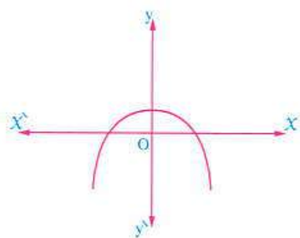
(39) Which of the following represents one - to - one function ?



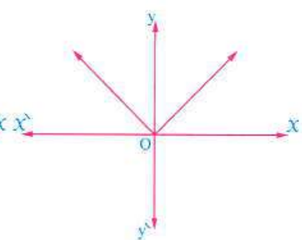

(a)




(b)

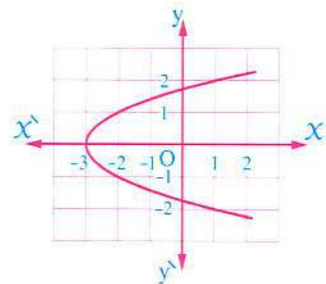



(c)




(d)

(40) The curve represented in the opposite figure is symmetric about the straight line whose equation is ...............




(a) $X = 0$ (b) $y = 0$

(c) $y = -2$ (d) $X = 2$

## Second Essay questions

**1** **In each of the following figures , mention the curve which is symmetric about the $X$-axis , the y-axis or the origin point :**

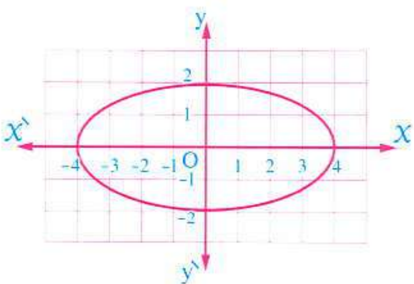



Fig. (1)

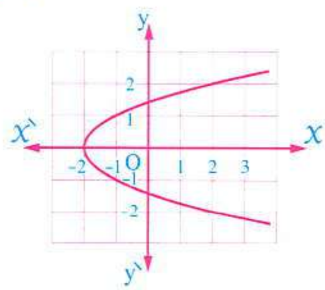



Fig. (2)

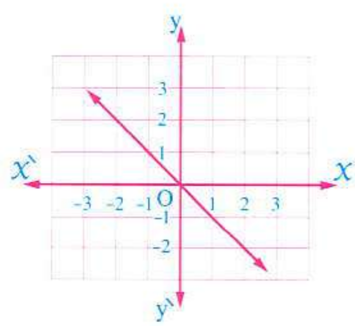



Fig. (3)

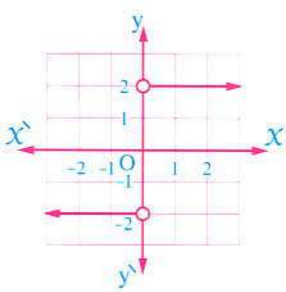



Fig. (4)

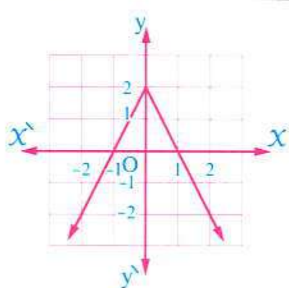



Fig. (5)

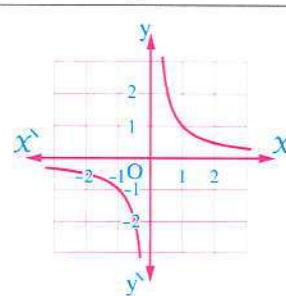
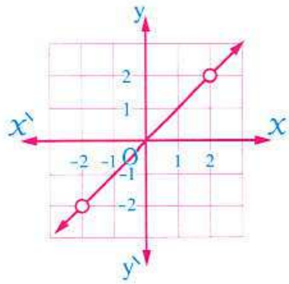


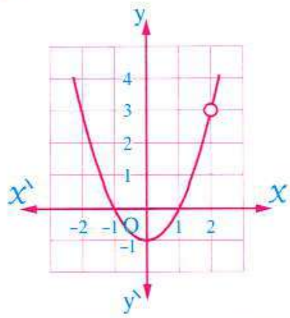

Fig. (6)

**2** Determine which of the functions represented by the following graphs is even , odd or neither even nor odd :

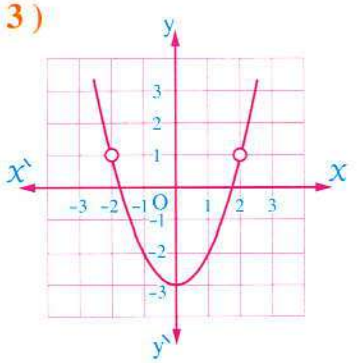
(1)



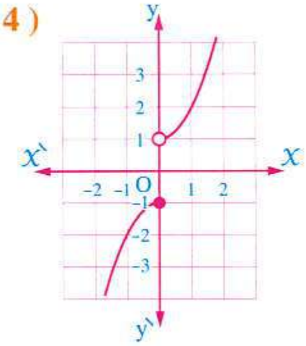
(2)



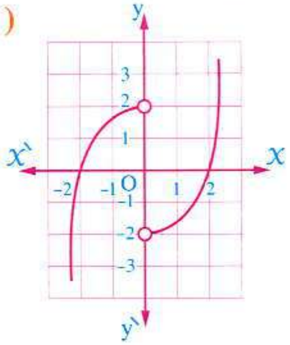
(3)



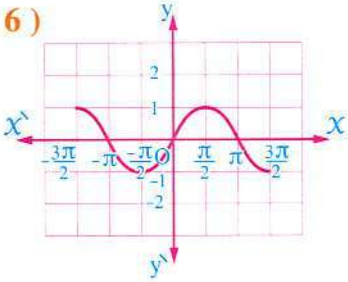
(4)



(5)



(6)



**3** Each of the following graphs represents the curve of the function $f$ , determine whether the function $f$ is even , odd or otherwise verifying your answers algebraically :

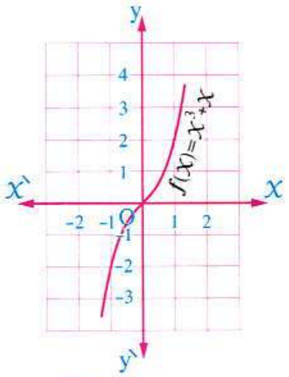



Fig. (1)

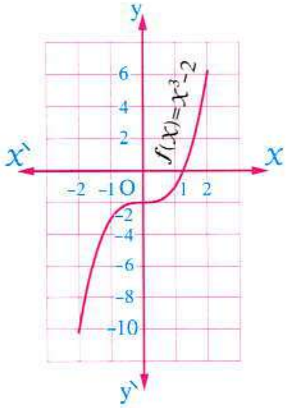



Fig. (2)

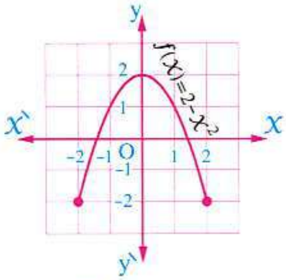



Fig. (3)

**4** Use the following figures to answer the following questions :

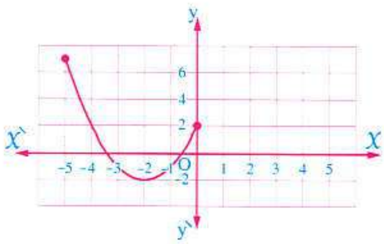


Fig. (1)

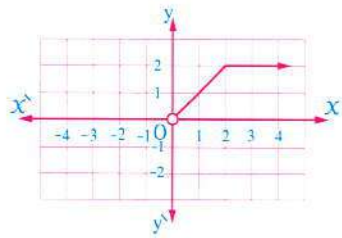


Fig. (2)

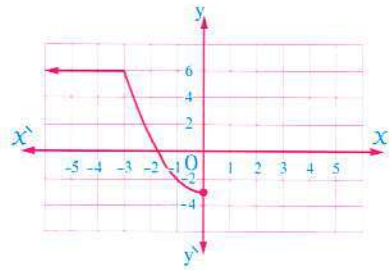


Fig. (3)

**First :** Complete the curve in each of fig. (1) and fig. (3) in your notebook to get an even function over its domain.

**Second :** Complete the curve in fig. (2) in your notebook to get an odd function over its domain.

**Third :** Determine the domain and the range of the function in each case , then show which graph represents a one - to - one function.

---

**5** **Determine which of the functions defined by the following rules is even , which is odd and which is neither even nor odd :**

(1) $f(x) = 5$

(2) $f(x) = x^4 + x^2 - 1$

(3) $f(x) = 3x - 4x^3$

(4) $f(x) = x^2 - 3x + 4$

(5) $f(x) = x^3(x^2 - 1)$

(6) $f(x) = (x-3)^2 - 7$

(7) $f(x) = \frac{x^2+2}{x-3}$

(8) $f(x) = \frac{2x^3 - x^5}{x}$

(9) $f(x) = \sqrt{x+3}$

(10) $f(x) = (x^2+1)^3$

(11) $f(x) = \sqrt{x^2+6}$

(12) $f(x) = \sqrt[3]{x^3 + x}$

(13) $f(x) = x^3 - \frac{1}{x}$

(14) $f(x) = \left(x - \frac{2}{x}\right)^3$

(15) $f(x) = \left(\frac{x^2}{3} - \frac{5}{x^4}\right)^5$

(16) $f(x) = \left(\frac{1-x}{1+x}\right)^2 - \left(\frac{1+x}{1-x}\right)^2$

(17) $f(x) = \left(\frac{x-1}{x+1}\right)^5 + \left(\frac{x+1}{x-1}\right)^5$

(18) $f(x) = (x^3+1)^4 - (x^3-1)^4$

(19) $f(x) = x \cos x$

(20) $f(x) = \frac{3x}{\tan x}$

(21) $f(x) = \frac{x^3 \sin 3x}{1+x^4}$

(22) $f(x) = \frac{3x^2 - \cos x}{x^3 - 6x}$

**(23)** $f(x) = x^2 \sin^3 x$

**(24)** $f(x) = x \sin x^3$

**(25)** $f(x) = \dfrac{x^2 + \tan x}{x^4 + \sin x}$

**(26)** $f(x) = \sin x^2 - \sin^2 x$

**(27)** $f(x) = x^4 + \sin^6 x$

**(28)** $f(x) = x^7 + \tan^5 x$

**(29)** $f(x) = \dfrac{\sin 3x \cos 2x}{\sec x}$

**(30)** $f(x) = 4^x + \cos x + 2^{-2x}$

**(31)** $f(x) = (\cos x + \sin x - 1)(\cos x + \sin x + 1)$

**(32)** $f(x) = \begin{cases} 2x & , x \geq 0 \\ -2x & , x < 0 \end{cases}$

**(33)** $f(x) = \begin{cases} 2x + x^2 & , x \leq 0 \\ 2x - x^2 & , x > 0 \end{cases}$

**6** **Each of the following graphs represents the curve of the function $f$ , show from the graph that the function $f$ is one - to - one verifying that algebraically :**

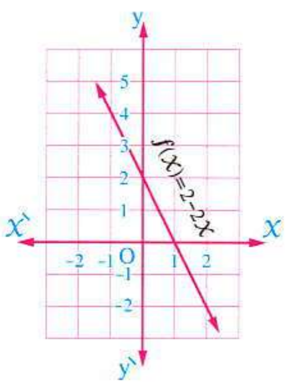


Fig. (1)

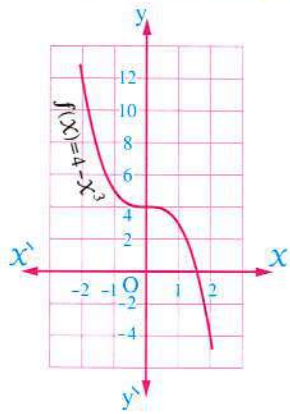


Fig. (2)

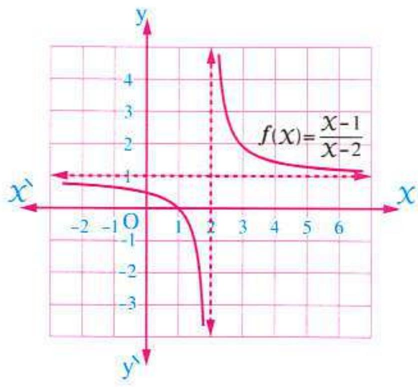


Fig. (3)

**7** **Prove that the functions that are defined by the following rules are one - to - one :**

**( 1 )** $f(x) = 2x - 3$

**( 2 )** $f(x) = 4 - x^3$

**( 3 )** $f(x) = \dfrac{3}{2x + 5}$

**( 4 )** $f(x) = \dfrac{3x - 5}{4x + 3}$

**8** **Prove that the functions that are defined by the following rules are not one - to - one :**

**( 1 )** $f(x) = 3$

**( 2 )** $f(x) = (x + 3)^2$

**( 3 )** $f(x) = x^2 - 5x + 6$

**( 4 )** $f(x) = \dfrac{1}{x^2 - 4}$

**9** **In each of the functions that are defined by the following rules determine whether the function is one - to - one or not giving reason :**

(1) $f(x) = 2x^2 - x - 3$

(2) $f(x) = x^4 + 2x^2 + 1$

**10** **Determine which of the following functions is even , odd or otherwise :**

(1) $f : \mathbb{R} \longrightarrow \mathbb{R}$ , $f(x) = x + 2$

(2) $f : [-3, 3[ \longrightarrow \mathbb{R}$ , $f(x) = 3x^2$

(3) $f(x) = x^2$ , $f : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$

(4) $f : f(x) = x^2$ , $x \in \mathbb{R} - \{3\}$

**11** **Let $f$ be a function whose domain is $\mathbb{R}$ find the value of $\frac{7f(-5) + 3f(5)}{2f(-5)}$ if :**

(1) $f$ is an odd function.

(2) $f$ is an even function. « 2 , 5 »

**12** **Let $f_1$ , $f_2$ , $g_1$ and $g_2$ be real functions such that :**

**$f_1(x) = x^4$ , $f_2(x) = \cos^5 x$ , $g_1(x) = 2x^3$ and $g_2(x) = \sin^3 x$**

**Determine which of the following functions is even , odd or otherwise :**

(1) $f_1 + g_2$

(2) $f_1 - f_2$

(3) $g_1 + g_2$

(4) $f_1 \times g_2$

(5) $g_1 \times g_2$

(6) $\frac{f_2}{f_1}$

**13** **Creative thinking : Represent graphically the curve which satisfies each of the following conditions :**

(1) Passes through the points (0 , – 2) , (2 , 2) , (3 , 7) and represents an even function.

(2) Passes through the points (0 , 0) , (– 2 , 1) , (– 3 , 5) and represents an odd function.

## Third Higher skills

**Choose the correct answer from those given :**

(1) If $f$ is a one - to - one function and the point (2 , 3) belongs to the function $f$ , which of the following points can be belong to $f$ ?

(a) (5 , 3) (b) (2 , – 1) (c) (3 , 2) (d) all the previous.

(2) If $f$ is a one - to - one function and the two points (2 , a) , (3 , b) belong to the function $f$ , which of the following is always true ?

(a) a > b (b) a = b (c) a ≠ b (d) a + b = 0

( 3 ) If $f$ is an odd function and $f(1) = k$ and $f(X + 2) = f(X) + f(2)$

, then $f(3) =$ ..............

(a) zero. (b) 3 k (c) 6 k (d) 9 k

( 4 ) If $f(X) = \frac{1 + X}{1 - X}$ and $g(X) = \frac{1 - X}{1 + X}$ , then each of the sum of these two functions and their product is .............. function.

(a) an even (b) an odd

(c) a one - to - one (d) neither even nor odd

( 5 ) If $f$ is a real function and $X$ , $-X \in$ the domain of the function , then the function $g(X) = f(X) + f(-X)$ is always ..............

(a) odd. (b) even.

(c) neither even nor odd. (d) one - to - one.

( 6 ) If $f$ is defined on $\mathbb{R}$ and $3 f(X) + 2 f(-X) = X^3 - \sin X$ , then $f$ is ..............

(a) odd. (b) even.

(c) neither even nor odd. (d) not one - to - one.

( 7 ) If $f(X) = X^3$ , $g(X) = X^2 + 1$ , which of the following is an odd function ?

(1) $(f \times g)$ (2) $(f \circ g)$ (3) $(g \circ f)$

(a) (1) only. (b) (2) , (3) (c) (1) , (2) (d) (1) , (3)



# Exercise 4

## Graphical representation of basic functions and graphing piecewise functions

From the school book · Understand · Apply · Higher Order Thinking Skills

Test yourself

### First Multiple choice questions

**Choose the correct answer from those given :**

( 1 ) If $f(X) = 5$ , then the domain of the function $f$ is ...............

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\{5\}$ (d) $\mathbb{R} - \{5\}$

( 2 ) If $f(X) = 7$ , then the range of the function $f$ is ...............

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\{7\}$ (d) $\mathbb{R} - \{7\}$

( 3 ) The range of the function $f : f(X) = \begin{cases} 0 & \text{when } X \le 0 \\ 1 & \text{when } X > 0 \end{cases}$ is ...............

(a) $\{1\}$ (b) $\{0\}$ (c) $\mathbb{R}$ (d) $\{0 , 1\}$

( 4 ) **In the opposite figure :**

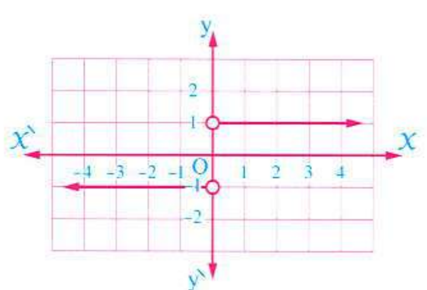
The range of the function is ...............

(a) $\{1\}$

(b) $\{1 , -1\}$

(c) $\{-1\}$

(d) $\mathbb{R}$




( 5 ) The range of the function $f : f(X) = \frac{3X^2 - 3}{X^2 - 1}$ is ...............

(a) $\mathbb{R} - \{1 , -1\}$ (b) $\mathbb{R} - \{3 , -3\}$ (c) $\{3 , -3\}$ (d) $\{3\}$

( 6 ) The range of the function $f : f(x) = \frac{x^2 - 3x}{x}$ where $x \neq 0$ is ...............

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$ (c) $\{0\}$ (d) $\mathbb{R} - \{-3\}$

( 7 ) The range of the function $f : f(x) = \frac{2x^3 - 2x}{x^2 - 1}$ is ...............

(a) $\mathbb{R} - \{1 , -1\}$ (b) $\mathbb{R} - \{2 , -2\}$ (c) $\mathbb{R}^+$ (d) $\mathbb{R}^+ - \{2\}$

( 8 ) The range of the function $f : [-2 , 3[ \longrightarrow \mathbb{R} , f(x) = x^2$ is ...............

(a) $[4 , 9[$ (b) $\mathbb{R}^+$ (c) $[0 , 9[$ (d) $[0 , 4]$

( 9 ) The range of the function $f : f(x) = \begin{cases} x & , x > 0 \\ -2 & , x \leq 0 \end{cases}$ is ...............

(a) $\mathbb{R}^+$ (b) $\mathbb{R}^+ - \{-2\}$ (c) $\mathbb{R}^+ \cup \{-2\}$ (d) $\mathbb{R}$

(10) The function $f$ where $f(x) = \begin{cases} 2 & , x > 0 \\ -2 & , x < 0 \end{cases}$ is symmetric about the point ...............

(a) $(2 , 0)$ (b) $(-2 , 0)$ (c) $(0 , 0)$ (d) $(2 , -2)$

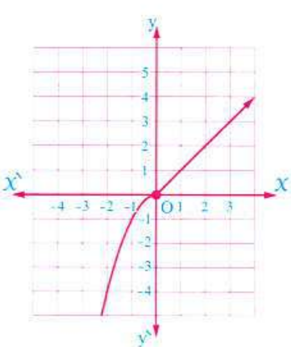
(11) The axis of symmetry for the function $f : f(x) = x^2$ is the straight line ...............

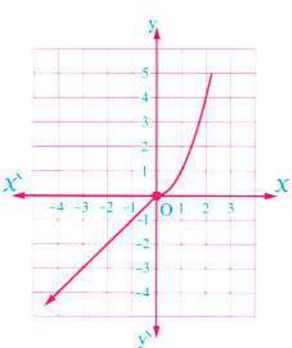
(a) $y = 0$ (b) $y = x$ (c) $y = -x$ (d) $x = 0$

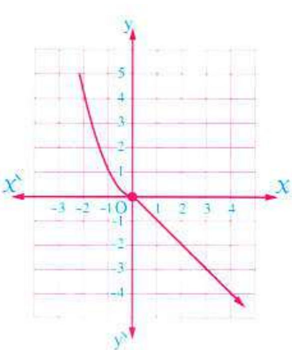
(12) The function $f : f(x) = \begin{cases} -x^2 & , x < 0 \\ \frac{1}{x} & , x > 0 \end{cases}$ is increasing on ...............

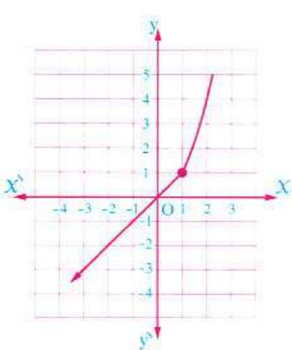
(a) $\mathbb{R}$ (b) $\mathbb{R}^-$ (c) $\mathbb{R}^+$ (d) $\mathbb{R} - \{0\}$

(13) The curve of the function $f : f(x) = \begin{cases} x^2 & , x > 0 \\ x & , x \leq 0 \end{cases}$ is ...............

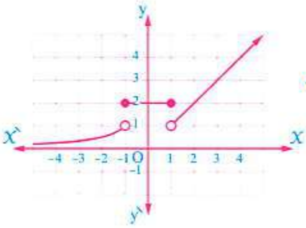

(a)

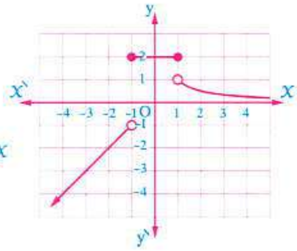

(b)

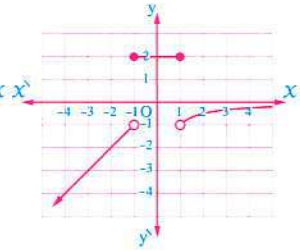

(c)

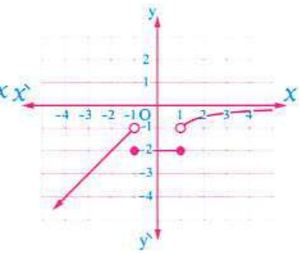

(d)

**(14)** The curve of the function $f : f(x) = \begin{cases} x & , \quad x < -1 \\ 2 & , \quad -1 \le x \le 1 \\ \frac{1}{x} & , \quad x > 1 \end{cases}$ is ...............

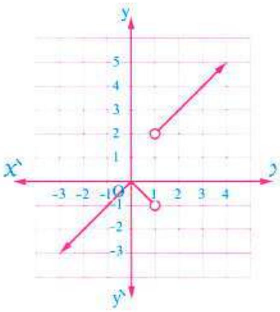

(a)

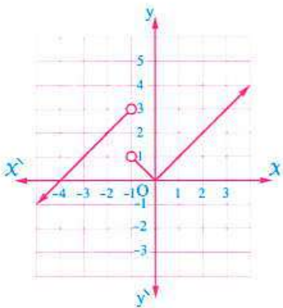

(b)

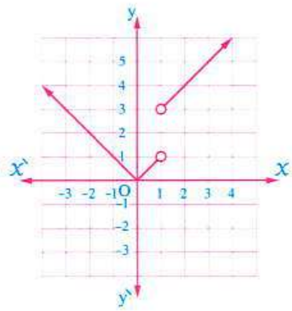

(c)

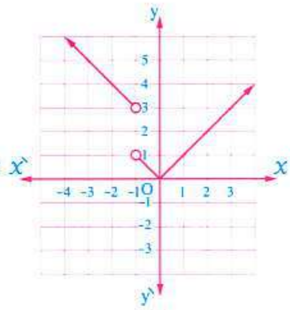

(d)

**(15)** The curve of the function $f : f(x) = \begin{cases} |x| & , \quad x < 1 \\ x + 2 & , \quad x > 1 \end{cases}$ is ...............


(a)


(b)


(c)


(d)

**(16) In the opposite figure :**

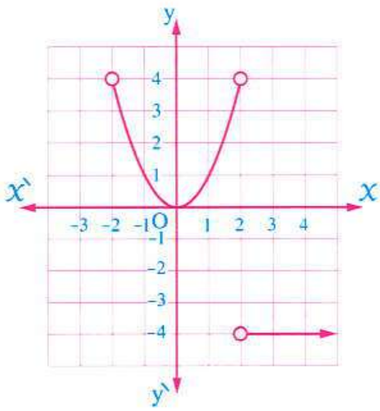
The curve of the function $f$ defined by the rule $f(x) =$ ...............

(a) $\begin{cases} x^2 & , \quad -2 < x < 2 \\ -4 & , \quad x < -2 \end{cases}$

(b) $\begin{cases} x^2 & , \quad -2 < x < 2 \\ -4 & , \quad x > 2 \end{cases}$

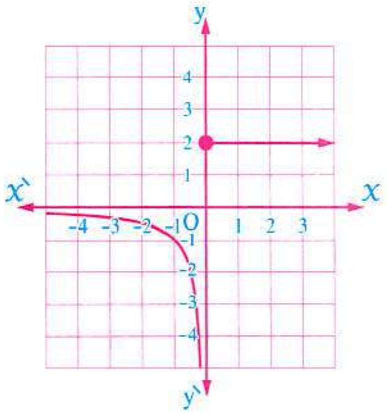
(c) $\begin{cases} x^2 & , \quad -2 \le x \le 2 \\ -4 & , \quad x < 2 \end{cases}$

(d) $\begin{cases} x^2 & , \quad -2 < x < 2 \\ -4 & , \quad x < 2 \end{cases}$



**(17) In the opposite figure :**

The curve of the function $f$ defined by the rule $f(x) = \cdots\cdots$




(a) $\begin{cases} 2 & , x > 0 \\ \frac{1}{x} & , x < 0 \end{cases}$

(b) $\begin{cases} 2 & , x \geq 0 \\ \frac{1}{x} & , x < 0 \end{cases}$

(c) $\begin{cases} 2 & , x < 0 \\ \frac{1}{x} & , x > 0 \end{cases}$

(d) $\begin{cases} 2 & , x \geq 2 \\ \frac{1}{x} & , x < 2 \end{cases}$

## Second Essay questions

**1 Graph each of the following functions and determine its range :**

(1) $f : \{-3, -1, 1, 2\} \longrightarrow [-3, 7]$ , $f(x) = 2x + 3$

(2) $g : [1, 5[ \longrightarrow \mathbb{R}$ , $g(x) = x + 1$

(3) $g : ]-\infty, -1[ \longrightarrow \mathbb{R}$ , $g(x) = 1 - x$

(4) $f : f(x) = -3x + 7$ for every $x \in \mathbb{R}$

(5) $f : \mathbb{R}^- \longrightarrow \mathbb{R}$ , $f(x) = x^2$

(6) $f : f(x) = x^2 - 5$ , $x \geq 0$

(7) $f : f(x) = x^2 - 3x$ , $x \in [-3, 2]$

**2** If $f : [-2, 6] \longrightarrow \mathbb{R}$ where $f(x) = \begin{cases} 4 - x & , -2 \leq x < 1 \\ x & , 1 \leq x \leq 6 \end{cases}$

(1) Graph the function $f$ , and from the graph deduce its range , and discuss its monotonicity.

(2) Is $f$ one - to - one ? Explain your answer.

**3** **Graph each of the functions defined by the following rules and from the graph. Find the domain and the range of each function and discuss its monotonicity and its type whether the function is even , odd or otherwise showing its symmetry :**

(1) $f(X) = \frac{3X^2 - 3}{X^2 - 1}$

(2) $f(X) = \frac{4 - X^2}{X + 2}$

(3) $f(X) = \frac{X^3 - X}{X^2 - 1}$

(4) $f(X) = \frac{X^4 - X^2}{X^2 - 1}$

**4** **Represent graphically each of the functions that are defined by the following rules , from the graph find the domain and the range of each function and discuss its monotonicity and its type whether it is even , odd or otherwise and show its symmetry :**

(1) $f : ]-\infty , 3[ \longrightarrow \mathbb{R}$ where $f(X) = 2$

(2) $f(X) = \begin{cases} 2 & , X \leq 0 \\ -3 & , X > 0 \end{cases}$

(3) $f(X) = \begin{cases} 2 & , X > 1 \\ X - 2 & , X \leq 1 \end{cases}$

(4) $f(X) = \begin{cases} X + 2 & , X \in [-2 , 1] \\ -X + 4 & , X \in ]1 , 4] \end{cases}$

(5) 📖 $f(X) = \begin{cases} 4 & , X < -2 \\ X^2 & , X \geq -2 \end{cases}$

(6) 📖 $f(X) = \begin{cases} X^2 & , X < 0 \\ X & , X \geq 0 \end{cases}$

(7) 📖 $f(X) = \begin{cases} X^3 & , X < 1 \\ 1 & , X > 1 \end{cases}$

(8) $f(X) = \begin{cases} X^3 & , X < 1 \\ 2 - X & , X \geq 1 \end{cases}$

(9) 📖 $f(X) = \begin{cases} |X| & , X \leq 0 \\ \frac{1}{X} & , X > 0 \end{cases}$

(10) 📖 $f(X) = \begin{cases} |X| & , X \leq 0 \\ X^2 & , X > 0 \end{cases}$

(11) $f(X) = \begin{cases} 3 & , X \leq -3 \\ |X| & , -3 < X < 3 \\ 3 & , X \geq 3 \end{cases}$

(12) $f(X) = \begin{cases} \frac{3X - 6}{2 - X} & , X > 2 \\ 2 & , X < 2 \end{cases}$

(13) $f(X) = \begin{cases} 2 & , -3 \leq X \leq -1 \\ 0 & , -1 < X < 1 \\ 2 & , 1 \leq X \leq 3 \end{cases}$

(14) $f(X) = \begin{cases} -X - 1 & , -4 \leq X < -2 \\ 1 & , -2 \leq X \leq 2 \\ X - 1 & , 2 < X \leq 4 \end{cases}$

**5** 📖 If $f_1 : \mathbb{R} \longrightarrow \mathbb{R}$ where $f_1(X) = 3X - 1$ , $f_2 : [-2 , 3] \longrightarrow \mathbb{R}$ where $f_2(X) = 3 - 2X$ , graph the function $(f_1 + f_2)$ showing its domain , then deduce its monotonicity.

**6** If $f(X) = X^3 - 4X$ , $g(X) = X^2 - 4$ , determine the domain of the function $\frac{f}{g}$ and graph it and from the graph determine its range and mention whether it is even , odd or otherwise , and discuss its monotonicity , and mention whether it is one - to - one or not.

# Exercise 12



## Introduction to limits of functions "Evaluation of the limit numerically and graphically"

📖 From the school book ● Understand ○ Apply ● Higher Order Thinking Skills

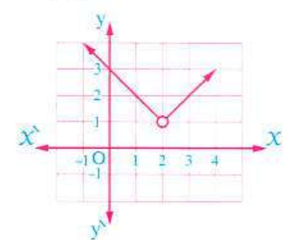
## First Multiple choice questions

**Choose the correct answer from those given :**

**( 1 )** All the following are unspecified quantities except ...............

(a) zero ÷ zero (b) $\infty - \infty$ (c) $\infty + \infty$ (d) $\infty \div \infty$

**( 2 ) In the opposite figure :**

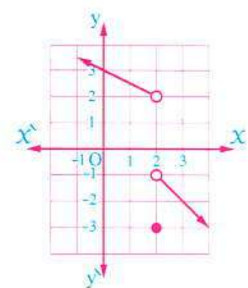


$\lim\limits_{x \to 2} f(x) =$ ...............

(a) 1 (b) – 1

(c) does not exist. (d) 2

**( 3 ) In the opposite figure :**

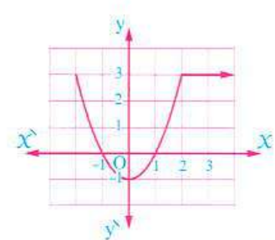


$\lim\limits_{x \to 2} f(x) =$ ...............

(a) – 3 (b) 2

(c) – 1 (d) does not exist.

**( 4 ) In the opposite figure :**

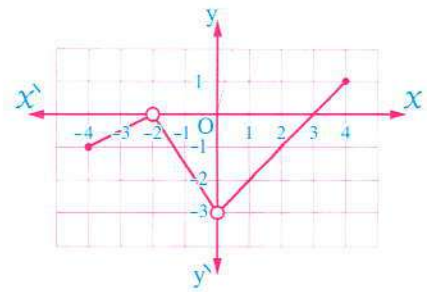


$\lim\limits_{x \to 2} f(x) =$ ...............

(a) zero (b) 2

(c) 3 (d) does not exist.

**(5) From the opposite figure :**



**First :** $\lim\limits_{x \to -2} f(x) =$ ..............

(a) zero (b) $-3$

(c) $-2$ (d) does not exist.

**Second :** $\lim\limits_{x \to 0} f(x) =$ ..............

(a) zero (b) $-2$ (c) $-3$ (d) does not exist.

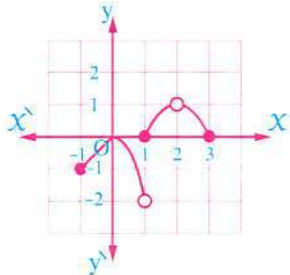
**Third :** $\lim\limits_{x \to -4} f(x) =$ ..............

(a) zero (b) $-4$ (c) $-1$ (d) does not exist.

**Fourth :** $\lim\limits_{x \to 4} f(x) =$ ..............

(a) zero (b) 4 (c) 1 (d) does not exist.

**(6) Using the opposite figure :**



**First :** $\lim\limits_{x \to 1^+} f(x) =$ ..............

(a) zero (b) $-2$

(c) 1 (d) does not exist.

**Second :** $\lim\limits_{x \to 1^-} f(x) =$ ..............

(a) zero (b) $-2$ (c) 1 (d) does not exist.

**Third :** $\lim\limits_{x \to 1} f(x) =$ ..............

(a) zero (b) $-2$ (c) 1 (d) does not exist.

**Fourth :** $\lim\limits_{x \to -1^+} f(x) =$ ..............

(a) zero (b) $-1$ (c) $-2$ (d) does not exist.

**Fifth :** $\lim\limits_{x \to -1} f(x) =$ ..............

(a) zero (b) $-1$ (c) $-2$ (d) does not exist.

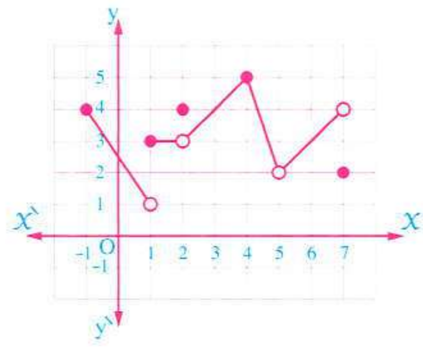
**Sixth :** $\lim\limits_{x \to 2} f(x) =$ ..............

(a) zero (b) 1 (c) 2 (d) does not exist.

**Seventh :** $\lim\limits_{x \to 0} f(x) =$ ..............

(a) zero (b) $-1$ (c) $-2$ (d) does not exist.

**(7) Using the opposite figure :**



**First :** $\lim\limits_{x \to -1^+} f(x) =$ ..............

(a) zero (b) $-1$

(c) 4 (d) does not exist.

**Second :** $\lim\limits_{x \to -1^-} f(x) =$ ..............

(a) zero (b) – 1 (c) 4 (d) does not exist.

**Third :** $\lim\limits_{x \to -1} f(x) =$ ..............

(a) zero (b) – 1 (c) 4 (d) does not exist.

**Fourth :** $f(2) =$ ..............

(a) zero (b) 3 (c) 4 (d) undefined.

**Fifth :** $f(5) =$ ..............

(a) zero (b) 2 (c) 5 (d) undefined.

**Sixth :** $\lim\limits_{x \to 5^-} f(x) =$ ..............

(a) zero (b) 2 (c) 3 (d) does not exist.

**Seventh :** $\lim\limits_{x \to 5^+} f(x) =$ ..............

(a) zero (b) 2 (c) 3 (d) does not exist.

**Eighth :** $\lim\limits_{x \to 5} f(x) =$ ..............

(a) zero (b) 2 (c) 3 (d) does not exist.

**Ninth :** $\lim\limits_{x \to 7^-} f(x) =$ ..............

(a) zero (b) 2 (c) 4 (d) does not exist.

**Tenth :** $\lim\limits_{x \to 7} f(x) =$ ..............

(a) zero (b) 2 (c) 4 (d) does not exist.

## Second Essay questions

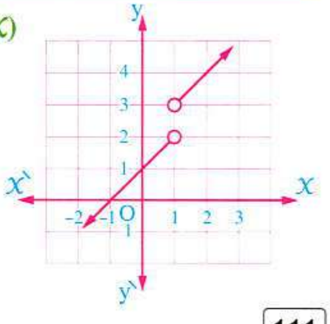
**1** Complete the following table and deduce $\lim\limits_{x \to 2} f(x)$ where $f(x) = 5x + 4$ :

| $x$ | 1.9 | 1.99 | 1.999 | ⟶ | 2 | ⟵ | 2.001 | 2.01 | 2.1 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | .......... | .......... | .......... | ⟶ | ? | ⟵ | .......... | .......... | .......... |

**2** Find each of the following limits graphically and numerically :

(1) $\lim\limits_{x \to 2} (1 - 3x)$ (2) $\lim\limits_{x \to 0} (x^2 - 2)$

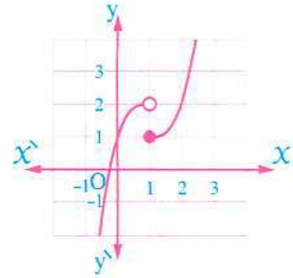
**3** If the opposite figure represents the curve of the function $f(x)$, find :




(1) $f(1)$ (2) $f(1^+)$

(3) $f(1^-)$ (4) $\lim\limits_{x \to 1} f(x)$

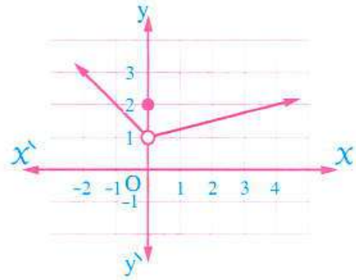
**4** **Study the opposite figure , then find :**




(1) $f(1)$ (2) $f(1^-)$

(3) $f(1^+)$ (4) $\lim_{x \to 1} f(x)$

**5** **Study the opposite figure which represents the function $f(x)$ , then find :**

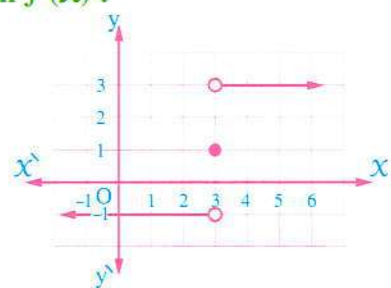



(1) $f(0)$ (2) $f(0^-)$

(3) $f(0^+)$ (4) $\lim_{x \to 0} f(x)$

(5) $f(2)$ (6) $\lim_{x \to 2} f(x)$

**6** **If the opposite figure represents the curve of the function $f(x)$ :**

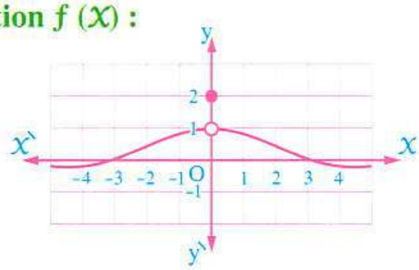



Find each of the following :

(1) $f(3)$ (2) $\lim_{x \to 3^-} f(x)$

(3) $\lim_{x \to 3^+} f(x)$ (4) $\lim_{x \to 3} f(x)$

**7** **If the opposite figure represents the curve of the function $f(x)$ :**

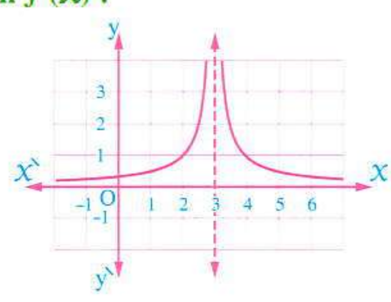



Find each of the following :

(1) $f(0)$ (2) $f(0^+)$

(3) $f(0^-)$ (4) $\lim_{x \to 0} f(x)$

**8** **If the opposite figure represents the curve of the function $f(x)$ :**



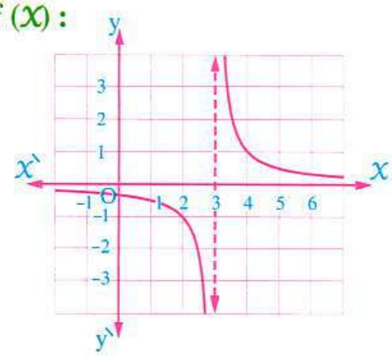

Find (if possible) each of the following :

(1) $f(3)$ (2) $\lim_{x \to 3^-} f(x)$

(3) $\lim_{x \to 3^+} f(x)$ (4) $\lim_{x \to 3} f(x)$

**9** **If the opposite figure represents the curve of the function $f(x)$ :**

Find (if possible) each of the following :



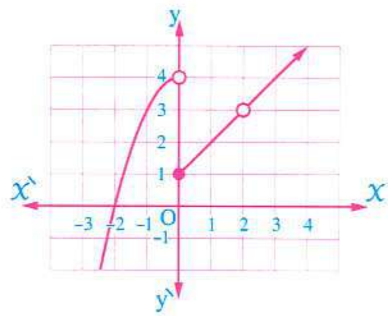

(1) $f(3)$ | (2) $f(3^+)$

(3) $f(3^-)$ | (4) $\lim\limits_{x \to 3} f(x)$

---

**10** **If the opposite figure represents the function $f(x)$**

**, find :**




(1) $f(0)$ | (2) $f(0^+)$

(3) $f(0^-)$ | (4) $\lim\limits_{x \to 0} f(x)$

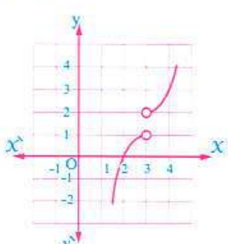
(5) $f(2)$ | (6) $f(2^+)$

(7) $f(2^-)$ | (8) $\lim\limits_{x \to 2} f(x)$

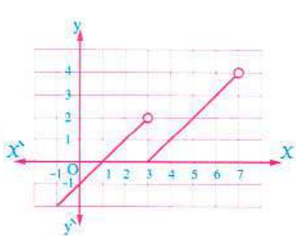
## Third Higher skills

**Choose the correct answer from those given :**

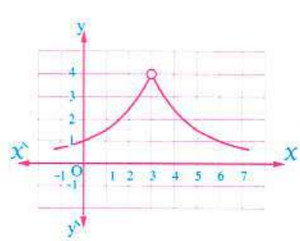
(1) Which of the functions represented by the following figures does have a limit at $x = 3$ ?



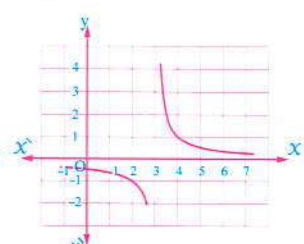
(a)



(b)



(c)



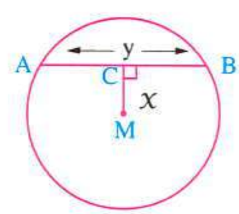
(d)

(2) The opposite figure represents circle M

, its radius length is 5 cm. , $\overline{MC} \perp \overline{AB}$

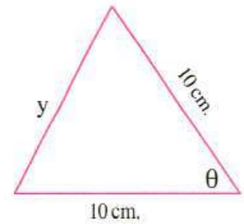
, the length of $\overline{MC} = x$ and the length of $\overline{AB} = y$

When $x \longrightarrow 0$ , then : $y \longrightarrow$ .............. cm.




(a) 2.5 (b) 5 (c) 10 (d) 20

**( 3 ) In the opposite figure :**




When $\theta \longrightarrow \frac{\pi}{2}$

, then : $y \longrightarrow$ ............... cm.

(a) 0 (b) 5 (c) 10 (d) $10\sqrt{2}$

( 4 ) If the curve of the polynomial function $f$ intersects the $X$-axis at $X = 3$ , then ............

(a) $\lim\limits_{X \to 3} f(X) = 0$ (b) $\lim\limits_{X \to 0} f(X) = 3$

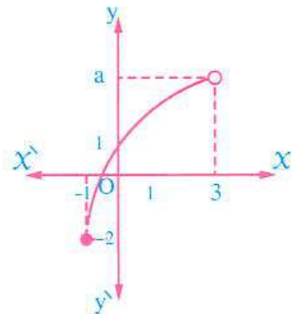
(c) $\lim\limits_{X \to 0} f(X) = 0$ (d) $\lim\limits_{X \to 3} f(X) = 3$

( 5 ) If the curve of the polynomial function $f$ intersects the y-axis at y = 3 , then ............

(a) $\lim\limits_{X \to 3} f(X) =$ zero (b) $\lim\limits_{X \to 3} f(X) = 3$

(c) $\lim\limits_{X \to 0} f(X) =$ zero (d) $\lim\limits_{X \to 0} f(X) = 3$

( 6 ) The opposite figure represents the curve of the function

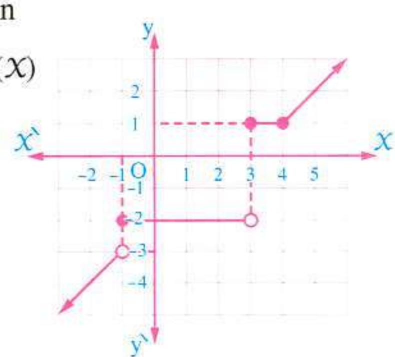



$y = f(X)$ and $\lim\limits_{X \to 0} f(X) = \lim\limits_{X \to -1^+} f(X) + \lim\limits_{X \to 3^-} f(X)$

, then a = ............

(a) 3 (b) 4 (c) 5 (d) 6

( 7 ) The opposite figure represents the curve of the function




$y = f(X)$ and $\lim\limits_{X \to a^-} f(X) = \lim\limits_{X \to -1^-} f(X) + \lim\limits_{X \to 3^+} f(X)$

, then the greatest possible value of a = ............

(a) 1 (b) 2

(c) 3 (d) 4

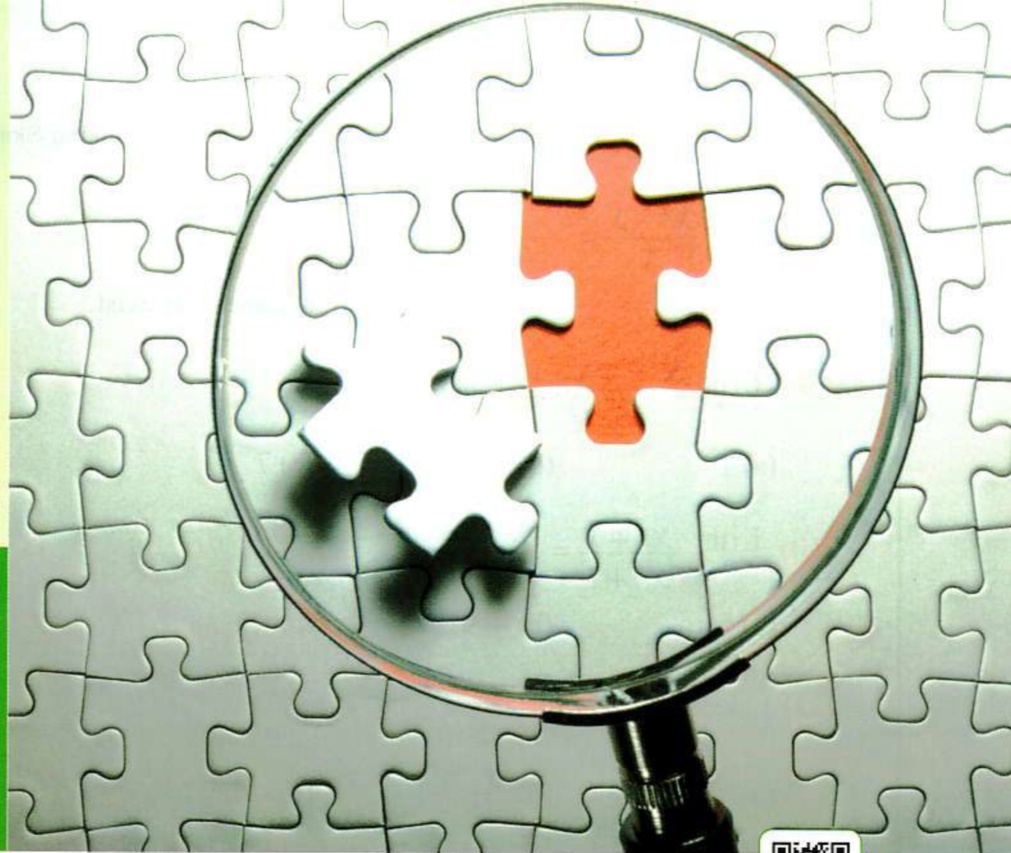




Exercise

# 13

## Finding the limit of a function algebraically



From the school book · Understand · Apply · Higher Order Thinking Skills

Test yourself

## First Multiple choice questions

**Choose the correct answer from the given ones :**

(1) $\lim\limits_{x \to \frac{1}{2}} (10) = \ldots\ldots$

(a) 5 (b) 20 (c) 10 (d) $10\frac{1}{2}$

(2) $\lim\limits_{x \to 4} \left(3x - \sqrt{x}\right) = \ldots\ldots$

(a) 8 (b) 10 (c) 14 (d) 16

(3) $\lim\limits_{x \to 2} (3a^2) = \ldots\ldots$

(a) 3 (b) 6 (c) $3a^2$ (d) 12

(4) $\lim\limits_{x \to a} (2x^2 - ax - a^2) = \ldots\ldots$ where $a \in \mathbb{R}$

(a) zero (b) a (c) $4a^2$ (d) $a^2 - 2a$

(5) $\lim\limits_{x \to \frac{\pi}{2}} (2x - \sin x) = \ldots\ldots$

(a) $\pi$ (b) $\pi + 1$ (c) $\pi - 1$ (d) $2\pi - 1$

(6) $\lim\limits_{x \to 2} \sqrt{\frac{3 + 2x}{4x - 1}} = \ldots\ldots$

(a) $-3$ (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(7) $\lim\limits_{x \to 3} \frac{2x - 6}{7x - 21} = \ldots\ldots$

(a) $\frac{2}{3}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) 3

**(8)** $\lim_{x \to 0} \frac{x^2 - x}{x} = \ldots\ldots\ldots\ldots$

(a) zero. (b) $-1$ (c) does not exist. (d) $1$

**(9)** $\lim_{x \to 3} \frac{x^2 - 7x + 12}{x - 3} = \ldots\ldots\ldots\ldots$

(a) $1$ (b) $-1$ (c) $7$ (d) $-2$

**(10)** $\lim_{x \to -1} \frac{x^2 + x}{x^3 + 1} = \ldots\ldots\ldots\ldots$

(a) zero. (b) $-\frac{1}{3}$ (c) $-1$ (d) has no existence.

**(11)** 📖 $\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 + x - 12} = \ldots\ldots\ldots\ldots$

(a) $\frac{5}{7}$ (b) $\frac{1}{7}$ (c) $-1$ (d) $-5$

**(12)** $\lim_{x \to -4} \frac{3x^2 - 48}{x + 4} = \ldots\ldots\ldots\ldots$

(a) $32$ (b) $-32$ (c) $24$ (d) $-24$

**(13)** $\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4} = \ldots\ldots\ldots\ldots$

(a) $\frac{4}{5}$ (b) $\frac{5}{4}$ (c) $\frac{2}{5}$ (d) $\frac{-2}{5}$

**(14)** $\lim_{x \to 0} \frac{3x + 2x^{-1}}{x + 4x^{-1}} = \ldots\ldots\ldots\ldots$

(a) $2$ (b) $4$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

**(15)** $\lim_{x \to \sqrt{5}} \frac{x^4 - x^2 - 20}{x - \sqrt{5}} = \ldots\ldots\ldots\ldots$

(a) $9$ (b) $2\sqrt{5}$ (c) $9\sqrt{5}$ (d) $18\sqrt{5}$

**(16)** $\lim_{x \to 4} \frac{(x - 3)^2 - 1}{x - 4} = \ldots\ldots\ldots\ldots$

(a) zero (b) $2$ (c) $3$ (d) $4$

**(17)** $\lim_{x \to 0} \frac{\sqrt{x + 1} - 1}{x} = \ldots\ldots\ldots\ldots$

(a) zero (b) $\sqrt{2}$ (c) $\frac{1}{2}$ (d) has no existence.

**(18)** $\lim_{x \to 3} \frac{\sqrt{x + 1} - 2}{x - 3} = \ldots\ldots\ldots\ldots$

(a) $4$ (b) $\frac{1}{4}$ (c) $6$ (d) $\frac{1}{6}$

**(19)** $\lim\limits_{x \to 9} \dfrac{\sqrt{2}-\sqrt{x-7}}{x-9} = \ldots\ldots$

(a) $2\sqrt{2}$ (b) $\dfrac{\sqrt{2}}{4}$ (c) $\dfrac{-\sqrt{2}}{4}$ (d) $-2\sqrt{2}$

**(20)** $\lim\limits_{x \to 1} \dfrac{4-\sqrt{x+15}}{1-x^2} = \ldots\ldots$

(a) 4 (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $-4$

**(21)** $\lim\limits_{x \to 4} \dfrac{\sqrt{x}-2}{5x^2-23x+12} = \ldots\ldots$

(a) $\frac{1}{92}$ (b) $\frac{1}{68}$ (c) 68 (d) 92

**(22)** $\lim\limits_{x \to 2} \dfrac{(x-3)^2-1}{\sqrt{x+2}-2} = \ldots\ldots$

(a) $-6$ (b) $-8$ (c) $-2$ (d) does not exist.

**(23)** $\lim\limits_{x \to 1} \dfrac{\sqrt{2x-1}-1}{\sqrt{3x+1}-2} = \ldots\ldots$

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{4}{3}$

**(24)** $\lim\limits_{x \to 1} \left(\dfrac{x^3}{x-1} - \dfrac{1}{x-1}\right) = \ldots\ldots$

(a) zero. (b) $-3$ (c) 3 (d) does not exist.

**(25)** $\lim\limits_{x \to 2} \dfrac{x^3-7x+6}{3x^2-8x+4} = \ldots\ldots$

(a) $\frac{5}{4}$ (b) $\frac{3}{2}$ (c) $\frac{4}{5}$ (d) $\frac{1}{3}$

**(26)** $\lim\limits_{x \to 2} \dfrac{3(x-2)^3+2x^2-8}{x-2} = \ldots\ldots$

(a) 0 (b) 2 (c) 4 (d) 8

**(27)** $\lim\limits_{x \to 0} \dfrac{7+2x}{\cos x} = \ldots\ldots$

(a) 7 (b) 8 (c) 9 (d) 1

**(28)** 📖 $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\tan x}{x} = \ldots\ldots$

(a) 0 (b) 1 (c) $\frac{4}{\pi}$ (d) does not exist.

**(29)** If $\lim\limits_{x \to 5} f(x) = 4$ , then $\lim\limits_{x \to 5} [f(x) - 4] = \ldots\ldots$

(a) 4 (b) zero. (c) 5 (d) 8

(30) $\lim\limits_{x \to 2} \frac{x-3}{x-2} = \dots\dots$

(a) $-1$ (b) $\frac{-3}{2}$ (c) $\frac{3}{2}$ (d) does not exist.

(31) If $\lim\limits_{x \to m} \frac{2x^2 - x - 3}{4x^2 - 9} = \frac{5}{12}$, then m = ……

(a) $\frac{3}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

(32) If $\lim\limits_{x \to 1} \frac{x^2 + 3x - 4}{x + a} = 5$, then a = ……

(a) $-1$ (b) $1$ (c) $0$ (d) $4$

(33) If $\lim\limits_{x \to 2} \frac{x^2 - 4a}{x - 2}$ exists, then a = ……

(a) $-1$ (b) $1$ (c) $2$ (d) $4$

(34) If $\lim\limits_{x \to 3} \frac{x^2 - 2x + k}{x^2 - 9} = m$, where $m \in \mathbb{R}$, then $k \times m = \dots\dots$

(a) $\frac{2}{3}$ (b) $-3$ (c) $-2$ (d) $-1$

(35) If $\lim\limits_{x \to -1} \frac{x^2 + kx + m}{x^2 - 1} = 3$, then k + m = ……

(a) $-4$ (b) $-5$ (c) $-8$ (d) $-9$

## Second Essay questions

**1 Find each of the following :**

(1) $\lim\limits_{x \to 5} \frac{x^2 - 25}{x - 5}$ « 10 »

(2) $\lim\limits_{x \to 3} \frac{x^2 - 8x + 15}{x - 3}$ « −2 »

(3) $\lim\limits_{x \to 0} \frac{x^2}{3x^3 - 2x^2}$ « $-\frac{1}{2}$ »

(4) $\lim\limits_{x \to 2} \frac{5x - 10}{4x - 8}$ « $\frac{5}{4}$ »

(5) $\lim\limits_{x \to 4} \frac{4x^2 - 64}{x - 4}$ « 32 »

(6) $\lim\limits_{x \to 4} \frac{2x - 8}{x^2 - x - 12}$ « $\frac{2}{7}$ »

(7) $\lim\limits_{x \to -3} \frac{x^2 + 4x + 3}{x^2 - 9}$ « $\frac{1}{3}$ »

(8) $\lim\limits_{x \to 2} \frac{x^2 - 2x}{x^2 - x - 2}$ « $\frac{2}{3}$ »

(9) $\lim\limits_{x \to -1} \frac{x^2 - 3x - 4}{x^2 - x - 2}$ « $\frac{5}{3}$ »

(10) $\lim\limits_{x \to \frac{1}{2}} \frac{2x^2 - 5x + 2}{2x - 1}$ « $\frac{-3}{2}$ »

(11) $\lim\limits_{x \to 9} \frac{9 - x}{x^2 - 81}$ « $-\frac{1}{18}$ »

(12) $\lim\limits_{x \to 1} \frac{3x^2 + x - 4}{x^2 - 1}$ « $\frac{7}{2}$ »

(13) $\lim\limits_{x \to \frac{3}{4}} \frac{16x^2 - 9}{8x - 6}$ « 3 »

(14) $\lim\limits_{x \to 3} \frac{2x^2 - 5x - 3}{2x^2 - 3x - 9}$ « $\frac{7}{9}$ »

(15) $\lim\limits_{x \to 3} \frac{x^2 + 2x - 15}{5x^2 - 45}$ « $\frac{4}{15}$ »

(16) $\lim\limits_{x \to 2} \frac{x^3 - 8}{3x^2 - 12}$ « 1 »

## 2 Find each of the following :

(1) $\lim\limits_{x \to 0} \frac{(x+2)^2-4}{x^2+x}$ «4»

(2) $\lim\limits_{x \to 0} \frac{(2x-1)^2-1}{5x}$ «$\frac{-4}{5}$»

(3) $\lim\limits_{x \to 2} \frac{(x-3)^2-1}{2x^2-3x-2}$ «$\frac{-2}{5}$»

(4) $\lim\limits_{x \to -3} \frac{(x+2)^3+1}{2x^2+6x}$ «$-\frac{1}{2}$»

(5) $\lim\limits_{x \to 2} \frac{x^4+x^2-20}{x-2}$ «36»

(6) $\lim\limits_{x \to 2} \frac{(x^2-4)^2}{x-2}$ «zero»

(7) $\lim\limits_{x \to 1} \frac{2x-2}{(x-1)^5+2x-2}$ «1»

(8) $\lim\limits_{x \to -2} \frac{x+2}{x^4-16}$ «$-\frac{1}{32}$»

(9) $\lim\limits_{x \to 9} \frac{x+\sqrt{x}-12}{x-9}$ «$\frac{7}{6}$»

(10) $\lim\limits_{x \to 0} \frac{\frac{1}{2+x}-\frac{1}{2}}{x}$ «$-\frac{1}{4}$»

(11) $\lim\limits_{x \to 2} \frac{x^4-2x^3-6x+12}{x^2+x-6}$ «$\frac{2}{5}$»

(12) $\lim\limits_{x \to 3} \left(\frac{5}{x}+\frac{x^2-3x}{x-3}\right)$ «$\frac{14}{3}$»

(13) $\lim\limits_{x \to -1} \left(\frac{x^2}{x^2-1}-\frac{3x+4}{x^2-1}\right)$ «$\frac{5}{2}$»

(14) $\lim\limits_{x \to 1} \left(\frac{1}{x-1}-\frac{3}{x^3-1}\right)$ «1»

## 3 Find each of the following :

(1) $\lim\limits_{x \to 4} \frac{x^3-15x-4}{x-4}$ «33»

(2) $\lim\limits_{x \to 1} \frac{x^3-2x+1}{x^2+x-2}$ «$\frac{1}{3}$»

(3) $\lim\limits_{x \to 4} \frac{x^4-21x^2+20x}{x-4}$ «108»

(4) $\lim\limits_{x \to -2} \frac{2x^3+3x^2+4}{x^3+8}$ «1»

(5) $\lim\limits_{x \to -2} \frac{x^2+4x+4}{x^3+x^2-8x-12}$ «$-\frac{1}{5}$»

(6) $\lim\limits_{x \to 1} \frac{x^3-5x^2+7x-3}{x^3-2x^2+x}$ «$-2$»

## 4 Find each of the following :

(1) $\lim\limits_{x \to 9} \frac{\sqrt{x}-3}{x-9}$ «$\frac{1}{6}$»

(2) $\lim\limits_{x \to 5} \frac{\sqrt{x-1}-2}{x-5}$ «$\frac{1}{4}$»

(3) $\lim\limits_{x \to -1} \frac{x+1}{\sqrt{x+5}-2}$ «4»

(4) $\lim\limits_{x \to 0} \frac{\sqrt{3-2x}-\sqrt{3}}{x}$ «$-\frac{1}{\sqrt{3}}$»

(5) $\lim\limits_{x \to 0} \frac{\sqrt{2x+9}-3}{x^2+x}$ «$\frac{1}{3}$»

(6) $\lim\limits_{x \to 5} \frac{x^2-5x}{\sqrt{x+4}-3}$ «30»

(7) $\lim\limits_{x \to 3} \frac{x^2-x-6}{\sqrt{5x-6}-3}$ «6»

(8) $\lim\limits_{x \to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$ «$\frac{1}{2}$»

(9) $\lim\limits_{x \to 1} \frac{x-1}{\sqrt{3x-2}-\sqrt{x}}$ « 1 »

(10) $\lim\limits_{x \to -1} \frac{\sqrt{x^2+8}-3}{x+1}$ « $-\frac{1}{3}$ »

(11) $\lim\limits_{x \to 1} \frac{1-x}{2-\sqrt{x^2+3}}$ « 2 »

(12) $\lim\limits_{x \to 3} \frac{\sqrt{x+1}-2}{\sqrt{x-2}-1}$ « $\frac{1}{2}$ »

(13) $\lim\limits_{x \to 4} \frac{\sqrt{24-5x}-2}{3-\sqrt{x+5}}$ « 7.5 »

(14) $\lim\limits_{x \to 3} \frac{x-3}{x-\sqrt{x+6}}$ « $\frac{6}{5}$ »

**5** If $\lim\limits_{x \to 2} \frac{f(x)-5}{x-2} = 1$ , then find : $\lim\limits_{x \to 2} f(x)$ « 5 »

**6** If $\lim\limits_{x \to 0} \frac{f(x)}{x^2} = 5$ **, then find :**

(1) $\lim\limits_{x \to 0} f(x)$

(2) $\lim\limits_{x \to 0} \frac{f(x)}{x}$ « zero , zero »

**7** If $\lim\limits_{x \to -1} \frac{x^2-(a-1)x-a}{x+1} = 4$ , then find a « –5 »

**8** If $\lim\limits_{x \to 1} \left(\frac{x^2+ax+b}{x-1}\right) = 5$

, find the value of each of a and b « 3 , –4 »

**9** **Trade :**

A company found that if it spent L.E. $x$ for advertising one of its production , then its profit will be given by the function $f : f(x) = 0.2x^2 + 40x + 150$

Find the profit of the company when the expenditure for advertisement approaches to L.E. 100 « 6150 pounds »

## Third Higher skills

**1** **Choose the correct answer from those givens :**

(1) If $f$ is a function satisfying that $x(f(x)+1) = f(x) + x^2$

, then $\lim\limits_{x \to 1} f(x) =$ ..............

(a) 1 (b) 2 (c) 3 (d) zero

(2) If $\lim\limits_{x \to 0} \frac{\sqrt{mx+k}-3}{x} = 2$ , then $\frac{m}{k} =$ ..............

(a) 8 (b) 4 (c) 3 (d) $\frac{4}{3}$

(3) If $\lim\limits_{X \to m} \left(2 f(X) - 5 g(X)\right) = 10$ , $\lim\limits_{X \to m} \left(f(X) + g(X)\right) = 6$

, then $\lim\limits_{X \to m} \frac{f(X)}{g(X)} = \ldots\ldots\ldots\ldots$

(a) $\frac{40}{7}$ (b) $\frac{2}{7}$ (c) 10 (d) 20

(4) If n (X) is a function and $\lim\limits_{X \to 2} \frac{n(X) - 8}{X - 2} = 7$

, then $\lim\limits_{X \to 2} \frac{2X^2 - n(X)}{X - 2} = \ldots\ldots\ldots\ldots$

(a) 1 (b) 7 (c) 8 (d) 15

**2** **Find each of the following :**

(1) $\lim\limits_{X \to 5} \left(\frac{X^2 - 4X - 5}{X^2 - 6X + 5}\right)^2$ «$\frac{9}{4}$»

(2) $\lim\limits_{X \to 1} \sqrt{\frac{X^2 + 2X - 3}{X^2 - X}}$ « 2 »

(3) $\lim\limits_{X \to \sqrt{2}} \frac{X^2 - 2}{X^2 + \sqrt{2}\, X - 4}$ «$\frac{2}{3}$»

(4) $\lim\limits_{X \to 1} \frac{5X^{-3} + 2X^{-1} - 7}{3X^{-2} + X^{-3} - 4}$ «$\frac{17}{9}$»

(5) $\lim\limits_{\tan X \to 3} \frac{\tan^2 X - 2\tan X - 3}{\tan^2 X - 4\tan X + 3}$ « 2 »

(6) $\lim\limits_{X \to 0} \frac{\cos^2 X - 5\cos X + 4}{\sin^2 X}$ «$\frac{3}{2}$»

(7) $\lim\limits_{(X-4) \to 4} \frac{\left(\sqrt[3]{X} - 2\right)\left(\sqrt[3]{X^2} + 2\sqrt[3]{X} + 4\right)}{X^2 - 64}$ «$\frac{1}{16}$»

(8) $\lim\limits_{X \to 1} \left(\frac{\sqrt{X^2 + 3} + 2}{X^2 - X} - \frac{2}{\sqrt{X^2 + 3} - 2}\right)$ « – 2 »

# Exercise 19

# The sine rule



From the school book | Understand | Apply | Higher Order Thinking Skills

Test yourself

## First Multiple choice questions

**Choose the correct answer from the given ones :**

**(1)** In any triangle XYZ , XY : YZ = ..............

(a) sin X : sin Y (b) sin Y : sin Z (c) sin Z : sin X (d) sin Z : sin Y

**(2)** In $\Delta$ ABC , if m ($\angle$ A) = 30° , C = $15\sqrt{3}$ cm. , m ($\angle$ C) = 60° , then a = .............. cm.

(a) 30 (b) 45 (c) 15 (d) 60

**(3)** DEF is a triangle in which m ($\angle$ D) = 80° and m ($\angle$ E) = 60° , if f = 12 cm.,
then d = .............. cm.

(a) $\frac{12 \sin 80°}{\sin 40°}$ (b) $\frac{12 \sin 80°}{\sin 60°}$ (c) $\frac{12 \sin 40°}{\sin 80°}$ (d) $\frac{12 \cos 80°}{\cos 40°}$

**(4)** In $\Delta$ ABC , if a = 4 cm. , b = 7 cm. , m ($\angle$ C) = 120° , then the area of the triangle = .............. $cm^2$.

(a) $7\sqrt{3}$ (b) $14\sqrt{3}$ (c) 7 (d) 14

**(5)** XYZ is an equilateral triangle , the length of its side is $10\sqrt{3}$ cm. , then the length of the diameter of its circumcircle is .............. cm.

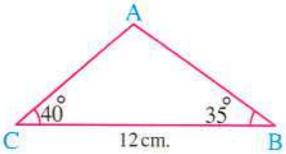
(a) 5 (b) 10 (c) 15 (d) 20

**(6)** In $\Delta$ XYZ , $\frac{x}{\sin X} = 6$ , then the length of the diameter of its circumcircle is .............. length units.

(a) 6 (b) 12 (c) 3 (d) 9

**( 7 ) In the opposite figure :**

The length of $\overline{AB} \approx$ .............. cm.



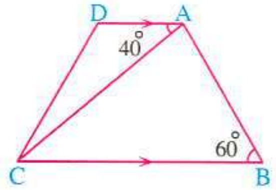

(a) 6 (b) 7

(c) 8 (d) 9

**( 8 ) In the opposite figure :**

$\overline{AD} \parallel \overline{BC}$ , AB = 4 cm. , m (∠ DAC) = 40° , m (∠ B) = 60°

, then the length of $\overline{AC} \simeq$ .............. cm.



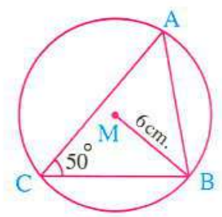

(a) 5 (b) 3

(c) 2 (d) 4

**( 9 ) In the opposite figure :**

M is the centre of the circle

BM = 6 cm. , then AB = .............. cm.




(a) 6 sin 50° (b) 12 sin 50°

(c) 6 cos 50° (d) 12 cos 50°

**(10)** A circle with diameter of length 20 cm. , passes through the vertices of Δ ABC which is an acute-angled triangle in which BC = 10 cm. , then m (∠ A) = ..............°

(a) 30 (b) 60 (c) 45 (d) 150

**(11)** In triangle ABC , m (∠ A) = 45° , the length of the radius of its circumcircle = 6 cm. , then a = .............. cm.

(a) 13 (b) $6\sqrt{2}$ (c) 12 (d) $\sqrt{2}$

**(12)** If the length of a side in any triangle = 12 cm. and the measure of the opposite angle to this side = 55° , then the circumference of the circle that passes through the vertices of this triangle $\simeq$ .............. cm.

(a) 36 (b) 42 (c) 46 (d) 52

**(13)** If the perimeter of triangle ABC equals 15 cm. , m (∠ A) = 53° , m (∠ B) = 47° , then the length of $\overline{AB} \simeq$ .............. cm.

(a) 6 (b) 7 (c) 5 (d) 8

**(14)** In triangle ABC , a = 27 cm. , m (∠ B) = 82° , m (∠ C) = 56° , then its surface area $\simeq$ .............. cm$^2$.

(a) 540 (b) 447 (c) 350 (d) 400

**(15)** In triangle ABC , m (∠ A) : m (∠ B) : m (∠ C) = 2 : 3 : 4 , AB = 12 cm. , then the length of $\overline{AC} \simeq$ .............. cm.

(a) 10 (b) 11 (c) 16 (d) 18

**(16)** In triangle ABC , which of the following statements is true ?

(a) sin A + cos B = a + b (b) a sin B = b sin A

(c) a = b sin c (d) $\frac{a}{\sin A} = \frac{\sin B}{b}$

**(17)** In Δ XYZ , 2 r sin X = .............. "where r is the radius length of its circumcircle"

(a) z (b) y (c) $x$ (d) area of Δ XYZ

**(18)** 📖 If r is the length of the radius of the circumcircle of the triangle XYZ , then $\frac{y}{2 \sin Y} =$ ..............

(a) r (b) 2 r (c) $\frac{1}{2}$ r (d) 4 r

**(19)** In acute-angled triangle ABC , $2a = \frac{b}{\sin B}$ , then m (∠ A) = ..............

(a) 30° (b) 45° (c) 60° (d) 75°

**(20)** In Δ ABC , sin A = 2 sin C , BC = 6 cm. , then AB = .............. cm.

(a) 2 (b) 3 (c) 4 (d) 6

**(21)** If the radius length of circumcircle of Δ ABC equals 3 cm. and sin A + sin B + sin C = 2, then the perimeter of triangle ABC = .............. cm.

(a) 6 (b) 9 (c) 12 (d) 24

**(22)** ABC is an equilateral triangle , its side length is 6 cm. and the area of its circumcircle equals k π $cm^2$ , then k = ..............

(a) $2\sqrt{3}$ (b) $8\sqrt{3}$ (c) 12 (d) 24

**(23)** In any triangle ABC , $\frac{\sin (A + B)}{\sin A + \sin B} =$ ..............

(a) 1 (b) $\frac{c}{a + b}$ (c) $\frac{a}{b + c}$ (d) $\frac{b}{a + c}$

**(24)** In Δ ABC , $\frac{a}{a + b} = \frac{\sin A}{\cdots\cdots}$

(a) sin B (b) sin C (c) sin A + sin B (d) sin A + sin C

**(25)** 📖 In Δ XYZ , if 3 sin X = 4 sin Y = 2 sin Z , then $x$ : y : z = ..............

(a) 2 : 3 : 4 (b) 6 : 4 : 3 (c) 3 : 4 : 6 (d) 4 : 3 : 6

(26) ABC is a triangle in which $\frac{\sin A}{3} = \frac{2 \sin B}{5} = \frac{\sin C}{4}$ , then a : b : c = ..............

(a) 6 : 5 : 8 (b) 8 : 5 : 6 (c) 7 : 2 : 4 (d) 3 : 5 : 4

(27) In Δ ABC : If $\frac{\sin A}{4} = \frac{\sin B}{9} = \frac{\sin C}{7}$ , then the greatest angle in measure is ..............

(a) ∠ A (b) ∠ B (c) ∠ C (d) right

(28) In triangle ABC , m (∠ A) : m (∠ B) : m (∠ C) = 3 : 5 : 4
, then $c^2 : a^2$ = ..............

(a) $\sqrt{6} : 2$ (b) 2 : 3 (c) 4 : 3 (d) 3 : 2

(29) In Δ ABC , $\frac{a}{b} \times \frac{\sin B}{\sin A}$ = ..............

(a) $\frac{c}{\sin C}$ (b) $\frac{\sin C}{c}$ (c) 4 r (d) 1

(30) In Δ ABC , if the radius of its circumcircle = 4 cm.
, then $\frac{a + b + c}{\sin A + \sin B + \sin C}$ = ..............

(a) 4 (b) 2 (c) 8 (d) 16

(31) If Δ ABC is a right-angled at ∠ B and b = 10 cm.
, then $\frac{a}{\sin A} + \frac{c}{\sin C}$ = .............. cm.

(a) 10 (b) 20 (c) 40 (d) 100

(32) If the radius of the circumcircle of Δ ABC equals r , then the perimeter of the triangle = .............. (sin A + sin B + sin C)

(a) r (b) 2 r (c) $4 r^2$ (d) $8 r^3$

(33) In Δ ABC , a – b = 4 cm. , sin A = $\frac{3}{2}$ sin B , then a = .............. cm.

(a) 4 (b) 6 (c) 8 (d) 12

(34) If the perimeter of Δ ABC is 24 cm. and sin A + sin B = 3 sin C , then C = .............. cm.

(a) 4 (b) 6 (c) 8 (d) 9

(35) ABC is a triangle , sin B + sin C = 4 sin A and b + c = 2 a + 10 cm.
, then a = .............. cm.

(a) 2 (b) 3 (c) 4 (d) 5

(36) In Δ ABC , AB = 8 cm. , BC = 12 cm. , m (∠ A) – m (∠ C) = 90°
, then tan C = ..............

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

**(37)** If r is the radius length of the circumcircle of $\Delta$ ABC and a = r , then m ($\angle$ A) = ..............

(a) 30° only. (b) 30° or 120° (c) 150° only. (d) 30° or 150°

**(38)** If the area of the triangle ABC is $\Delta$ and r is the radius length of the circumcircle of the triangle ABC , then : $\frac{4 r \Delta}{abc}$ = ..............

(a) 1 (b) 2 (c) 4 (d) 8

**(39)** In $\Delta$ ABC , $\frac{2 b}{\sin B}$ = .............. r (where r is the radius of its circumcircle)

(a) 1 (b) 2 (c) 4 (d) 8

**(40)** ABC is a triangle , b = 12 cm. , the radius length of its circumcircle is r , then the area of the triangle = .............. $cm^2$.

(a) $\frac{2 ac}{r}$ (b) $\frac{3 ac}{r}$ (c) $\frac{4 ac}{r}$ (d) $\frac{6 ac}{r}$

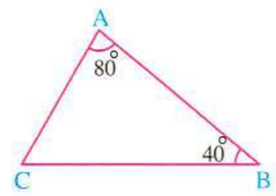
**(41)** If the triangle ABC is an isosceles right-angled triangle and r is the radius length of the circumcircle of the triangle ABC , then the area of $\Delta$ ABC = .............. (in terms of r)

(a) $\frac{1}{2} r^2$ (b) $2 r^2$ (c) $r^2$ (d) $4 r^2$

**(42)** **In the opposite figure :**

If the perimeter of $\Delta$ ABC = 20 cm. ,

then the diameter length of its circumcircle ≈ .............. cm.




(a) 2 (b) 4

(c) 6 (d) 8

**(43)** In $\Delta$ ABC , cos (B + C) = $\frac{3}{5}$ , BC = 8 cm. , then the radius length of the circumcircle of $\Delta$ ABC = .............. cm.

(a) 4 (b) 5 (c) 8 (d) 10

**(44)** If the area of a triangle is $\frac{a^2 \sin B \sin C}{k \sin A}$ , then k = .............. , k ≠ 0

(a) 1 (b) 2 (c) 3 (d) 4

**(45)** If r is the radius length of the circumcircle of triangle ABC and a = $\frac{1}{2}$ r , then m ($\angle$ A) = .............. where A is an acute angle.

(a) $\sin^{-1}(1)$ (b) $\sin^{-1}(2)$ (c) $\sin^{-1}\left(\frac{1}{2}\right)$ (d) $\sin^{-1}\left(\frac{1}{4}\right)$

(46) If Δ is the area of triangle XYZ , S is half the perimeter of the triangle XYZ , then $\frac{2}{z \sin X} + \frac{2}{x \sin Y} + \frac{2}{y \sin Z} = $ ..............

(a) $\frac{S}{\Delta}$ (b) $\frac{2S}{\Delta}$ (c) $\frac{3S}{\Delta}$ (d) $\frac{4S}{\Delta}$

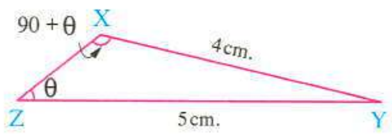
(47) In the opposite figure tan θ = ..............

(a) $\frac{3}{5}$

(b) $\frac{4}{3}$

(c) $\frac{5}{4}$

(d) $\frac{4}{5}$




## Second Essay questions

**1** XYZ is a triangle in which m (∠ X) = 80° , m (∠ Y) = 60° and z = 10 cm. , find each of $x$ and y to the nearest cm. « 15 cm. , 13 cm. »

**2** ABC is a triangle in which c = 19 cm. , m (∠ A) = 112° and m (∠ B) = 33°

Find to the nearest hundredth each of b and the length of the radius of the circumcircle of the triangle. « 18.04 cm. , 16.56 cm. »

**3** LMN is a triangle in which m = 68.4 cm. , m (∠ M) = 100° and m (∠ N) = 40°

, find : ( 1 ) $\ell$

( 2 ) The length of the radius of the circumcircle of the triangle LNM

( 3 ) The area of the triangle LMN « 44.64 cm. , 34.73 cm. , 981.34 cm$^2$. »

**4** LMN is a triangle in which m (∠ L ) = 18° 52′ , m (∠ N) = 44° 17′ and NL = 35 cm.

Find :

( 1 ) The length of each of $\overline{MN}$ and $\overline{LM}$

( 2 ) The area of the circumcircle of Δ LMN « 12.7 cm. , 27.4 cm. , 1208.67 cm$^2$. »

**5** ABC is a triangle in which b = 10 cm. , m (∠ A) = 40° and m (∠ C) = 80°

Find the length of the greatest side of Δ ABC « 11 cm. »

**6** ABC is a triangle in which c = 4.5 cm. , m (∠ A) = 100° and m (∠ B) = 15°

Find the length of the smallest side of Δ ABC « 1.3 cm. »

**7** ABC is a triangle in which $m(\angle A) = 60^\circ$ and $a = 7\sqrt{3}$ cm. Find the area and the circumference of the circumcircle of $\Delta ABC$ $\left(\pi = \frac{22}{7}\right)$ « 154 cm$^2$, 44 cm. »

---

**8** ABC is a triangle in which $m(\angle A) = 60^\circ$, $m(\angle B) = 45^\circ$

**Prove that :** $a : b : c = \sqrt{6} : 2 : \sqrt{3} + 1$

---

**9** ABC is a triangle in which : $a = 13$ cm., $m(\angle A) = 53^\circ\, 8'$, $c = 15$ cm. Find the radius length of the circumcircle of $\Delta ABC$, then find $m(\angle C)$ « 8.1 cm., $67^\circ\, 23'\, 9''$ or $112^\circ\, 36'\, 51''$ »

---

**10** ABC is a triangle in which $m(\angle A) = 35^\circ$, $a = 8$ cm. and $b = 6$ cm.

**Find :** $m(\angle B)$ « $25^\circ\, 28'\, 45''$ »

---

**11** Find the perimeter of the triangle ABC in which $c = 8.7$ cm., $m(\angle A) = 57^\circ\, 13'$ and $m(\angle B) = 64^\circ\, 18'$ « 26.5 cm. »

---

**12** ABC is a triangle in which $m(\angle B) = 45^\circ$, $m(\angle C) = 60^\circ$ and the diameter length of the circumcircle of $\Delta ABC = 40$ cm. Calculate the area and the perimeter of $\Delta ABC$ to the nearest whole number. « 473 cm$^2$, 102 cm. »

---

**13** ABC is an isosceles triangle in which : $m(\angle A) = 120^\circ$ and the length of the radius of the circumcircle of $\Delta ABC$ is 12 cm.

Find c and calculate the area of $\Delta ABC$ « 12 cm., 62.4 cm$^2$ »

---

**14** ABC is an isosceles triangle in which : $a = b$ and $m(\angle A) = 15^\circ$ and the perimeter of $\Delta ABC$ is 25 cm. Find the area of the circumcircle of $\Delta ABC$ « 474 cm$^2$ »

---

**15** If the perimeter of $\Delta ABC = 40$ cm., $m(\angle A) = 44^\circ$ and $m(\angle B) = 66^\circ$, Find the lengths of the sides of the triangle ABC « 10.9 cm., 14.3 cm., 14.8 cm. »

---

**16** ABC is a triangle in which $c = 12$ cm. and $m(\angle B) = 3\, m(\angle A) = 60^\circ$

Find a and the area of $\Delta ABC$ to the nearest cm$^2$ « 4.2 cm., 22 cm$^2$ »

---

**17** If the area of the triangle ABC is 450 cm$^2$, $m(\angle B) = 82^\circ$ and $m(\angle C) = 56^\circ$, find the value of a « 27 cm. »

---

**18** ABC is an acute-angled triangle in which AC = 12 cm., $\sin A = 0.6$ and its area is 43.2 cm$^2$

Find the length of each of $\overline{AB}$ and $\overline{BC}$, Also find $m(\angle B)$ « 12 cm., 7.6 cm., $71^\circ\, 34'$ »

---

**19** Find the perimeter of the acute-angled triangle ABC if $a = 7$ cm., $b = 8$ cm. and $m(\angle A) = 60^\circ$ « 20 cm. »

**20** ABC is a right-angled triangle at B , let $D \in \overrightarrow{BC}$ and $D \notin \overline{BC}$ such that CD = 4 cm. , m (∠ ADC) = 45° 15́ and m (∠ CAD) = 18°

Find the length of $\overline{AB}$ to the nearest cm. « 8 cm. »

---

**21** XYZ is a triangle in which YZ = 15 cm. , m (∠ Y) = 30° and m (∠ Z) = 70°

Calculate the length of the perpendicular dropped from X to $\overline{YZ}$ « 7.16 cm. »

---

**22** ABC is an obtuse-angled triangle at C in which : a = 8 cm. , c = 20 cm.

and $\tan A = \frac{1}{2\sqrt{2}}$ Find m (∠ C) « 123° 33́ »

---

**23** ABC is a triangle in which : b = 5 cm. , $\tan C = \frac{4}{3}$ and m (∠ B) = 30°

find a , c and the area of the triangle to the nearest integer. « 10 cm. , 8 cm. , 20 $cm^2$ »

---

**24** XYZ is a triangle in which sin X + sin Y + sin Z = 2.37 and its perimeter is 56.88 cm.

Find the length of the radius of the circumcircle of Δ XYZ « 12 cm. »

---

**25** ABC is a triangle in which m (∠ A) = 60° , m (∠ B) = 45° , if $a + b = (\sqrt{6} + 2)$ cm. , then find each of a and b « $\sqrt{6}$ cm. , 2 cm. »

---

**26** ABC is a triangle in which sin A : sin B : sin C = 2 : 4 : 5 and c – b = 3 cm.

Find each of a and b « 6 cm. , 12 cm. »

---

**27** ABC is a triangle in which m (∠ A) : m (∠ B) : m (∠ C) = 3 : 4 : 3 , if a = 5 cm. , then find the perimeter of the triangle ABC « 15.9 cm. »

---

**28** ABC is a triangle in which m (∠ A) : m (∠ B) : m (∠ C) = 1 : 3 : 5

Find the length of the smallest side of Δ ABC if its perimeter equals 16 cm. « 2.5 cm. »

---

**29** ABC is a triangle in which $m(\angle A) = \frac{2}{3} m(\angle B) = \frac{1}{2} m(\angle C)$ , the length of the radius of its circumcircle = 10 cm. Find the area of Δ ABC « 110 $cm^2$ »

---

**30** ABC is a triangle in which 6 sin A = 4 sin B = 3 sin C and its perimeter is 45 cm.

Find each of a and c « 10 cm. , 20 cm. »

---

**31** If ABC is a triangle where : $\frac{a+b-c}{3} = \frac{a+c-b}{5} = \frac{b+c-a}{7}$

, then prove that sin A : sin B : sin C = 4 : 5 : 6

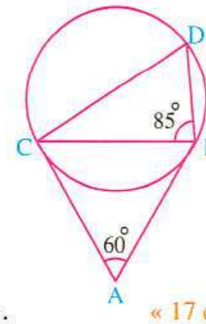
**32** **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent segments to the circle at B and C

, if m (∠ A) = 60° , m (∠ DBC) = 85°

and the area of the triangle ABC = $9\sqrt{3}$ cm²

, then find the perimeter of the triangle DBC to the nearest centimetre. « 17 cm.»




---

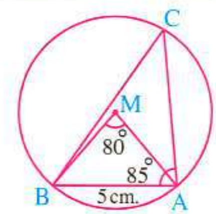
**33** **In the opposite figure :**

A circle M , AB = 5 cm. , m (∠ AMB) = 80°

and m (∠ CAB) = 85°

**Find : ( 1 )** The perimeter of Δ ABC

**( 2 )** The area of the circle M « 19.12 cm. , 47.5 cm² »




---

**34** ABCD is a parallelogram in which m (∠ A) = 50° , m (∠ DBC) = 70° and BD = 8 cm.

Find the perimeter of the parallelogram. « 38 cm. »

---

**35** ABCD is a parallelogram in which AB = 18 cm. , m (∠ CAB) = 36° and m (∠ DBA) = 44°

Find the length of the diagonal $\overline{AC}$ and the area of the parallelogram. « 25.39 cm. , 269 cm² »

---

**36** ABCD is a parallelogram. M is the point of intersection of its two diagonals.

Let AC = 20 cm. , m (∠ AMD) = 130° and m (∠ CAB) = 85°

Find the length of $\overline{BD}$ and the area of the parallelogram ABCD « 28.2 cm. , 216 cm² »

---

**37** ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$ , AD = 20 cm. , m (∠ D) = 120° ,

m (∠ B) = 62° and m (∠ ACB) = 23° 25′

**Find :**

**( 1 )** The length of each of $\overline{AC}$ and $\overline{BC}$ to the nearest cm.

**( 2 )** The area of the trapezium ABCD to the nearest cm². « 29 cm. , 33 cm. , 305 cm² »

---

**38** ABCD is a quadrilateral in which CD = 100 cm. , m (∠ BCA) = 36° ,

m (∠ BDA) = 55° , m (∠ BCD) = 85° and m (∠ CDA) = 87°

Find the lengths of $\overline{BD}$ and $\overline{AC}$ to the nearest centimetre. « 112 cm. , 144 cm. »

---

**39** ABCD is a quadrilateral in which m (∠ ABC) = 90° , m (∠ BAD) = 80°

, AB = AD = 10 cm. , BD = BC Calculate the area of the quadrilateral ABCD « 102 cm² »

**40** **In any triangle ABC , prove that :**

( 1 ) $\frac{3a - 4b}{3 \sin A - 4 \sin B} = \frac{c}{\sin C}$

( 2 ) The area of the triangle $= \frac{a^2 \sin B \sin C}{2 \sin A}$

( 3 ) The area of the triangle $= \frac{a b c}{4 r}$

"where r is the radius length of the circumcircle of the triangle ABC"

## Third Higher skills

**1** **Choose the correct answer from the given ones :**

( 1 ) If the radius length of the circumcircle of the triangle ABC equals 3 cm.

, then $\frac{abc}{\sin A \sin B \sin C} =$ ...............

(a) 3 (b) 6 (c) 27 (d) 216

( 2 ) If ABC is a triangle , then : a csc A + b csc B + c csc C = ...............

(a) 2r (b) 4r (c) 6r (d) 8r

( 3 ) If a = sin B , b = sin C , c = sin A , then the circumference of the circumcircle of Δ ABC equals ...............

(a) 1 (b) $\frac{\pi}{2}$ (c) $\pi$ (d) $2\pi$

( 4 ) In Δ ABC , $\frac{a \sin A + b \sin B + c \sin C}{a^2 + b^2 + c^2} =$ ...............

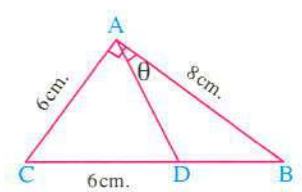
(a) $\frac{1}{r^2}$ (b) $\frac{1}{2r}$ (c) 2r (d) $r^2$

( 5 ) The area of Δ ABC is 24 $cm^2$ and the radius length of its circumcircle is 5 cm.

, then sin A sin B sin (A + B) = ...............

(a) $\frac{3}{25}$ (b) $\frac{6}{25}$ (c) $\frac{9}{25}$ (d) $\frac{12}{25}$

( 6 ) **In the opposite figure :**

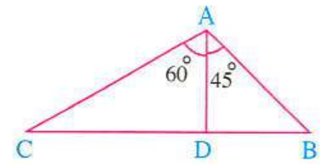



cot θ = ...............

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) 2 (d) 1

**(7) In the opposite figure :**

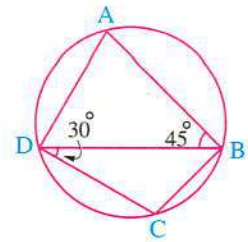



If CD = 2DB , then $\frac{\sin B}{\sin C}$ = ..............

(a) $\sqrt{6}$ (b) $\frac{\sqrt{6}}{4}$

(c) $\frac{\sqrt{6}}{3}$ (d) $\frac{2\sqrt{6}}{3}$

**(8) In the opposite figure :**

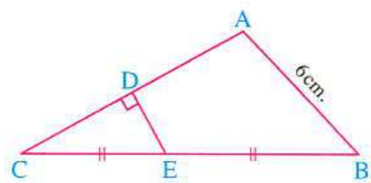



If AD = $4\sqrt{2}$ cm. , then BC = .............. cm

(a) $2\sqrt{2}$ (b) 4

(c) $4\sqrt{2}$ (d) 8

**(9) In the opposite figure :**



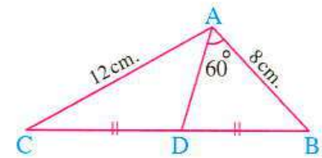

If tan (∠ DEC) = $\frac{3}{4}$ ,

then the radius length of

the circumcircle of Δ ABC = .............. cm.

(a) 3 (b) 3.5 (c) 3.75 (d) 4

**(10) In the opposite figure :**



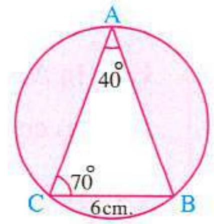

D is the midpoint of $\overline{BC}$ and

m (∠ BAD) = 60°

, then tan (∠ DAC) = ..............

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\sqrt{2}$

**(11) In the opposite figure :**




The area of the shaded part ≈ .............. $cm^2$.

(a) 4.37 (b) 26.2

(c) 43.7 (d) 52.6

**2** If ABCD is a cyclic quadrilateral , **prove that :** BC × sin (∠ ABD) = AD × sin (∠ CDB)

**3** **In the triangle ABC , prove that :**

**(1)** $\sin A + \sin B + \sin C = \frac{4 S \Delta}{a b c}$

**(2)** $\frac{1}{c \sin A} + \frac{1}{a \sin B} + \frac{1}{b \sin C} = \frac{S}{\Delta}$

where S is half of the triangle's perimeter and Δ is the triangle's area.

**4** Prove that the area of the circumcircle of Δ ABC equals $\frac{\pi a b}{4 \sin A \sin B}$

# FIRST Monthly tests of October

## Test 1

Total mark

20

**1 Choose the correct answer from those given :** (12 marks)

(1) $\lim\limits_{x \to 3} \frac{x^2 - 5x + 6}{x^2 - 9} =$ ..............

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{5}$ (d) $\frac{1}{4}$

(2) The domain of the function $f : f(x) = \frac{\sqrt{x-2}}{x-4}$ is ..............

(a) $[2, \infty[$ (b) $]-\infty, 2]$

(c) $[2, \infty[ - \{4\}$ (d) $]-\infty, 2] - \{4\}$

(3) All the following functions is one - to - one on its domain except $f(x) =$ ..............

(a) $3x$ (b) $\frac{1}{x}$ (c) 5 (d) $x^3$

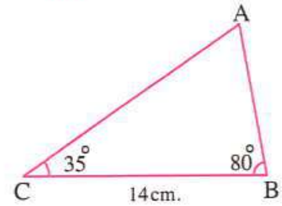
(4) If the radius of the circumcircle of triangle ABC equals 3 cm.

and $\sin A + \sin B + \sin C = 2$ , then the perimeter of the triangle = .............. cm.

(a) 6 (b) 9 (c) 12 (d) 24

(5) The greatest side in the given triangle ≃ .............. cm.

(to the nearest integer)




(a) 20 (b) 16

(c) 15 (d) 14

(6) If $\lim\limits_{x \to 1} \frac{x^2 - k}{x + 2} = -1$ , then k = ..............

(a) 4 (b) $-2$ (c) 2 (d) $\pm 2$

(7) The function $f : f(x) = |x + 2|$ is decreasing on the interval ..............

(a) $]0, \infty[$ (b) $[2, \infty[$ (c) $]-2, \infty[$ (d) $]-\infty, -2[$

(8) If $f$ is an odd function , then $\frac{7f(3) + 3f(-3)}{2f(-3)} =$ ..............

(a) 2 (b) $-2$ (c) 5 (d) $-0.5$

(9) If $f(x) = x - 3$ , $g(x) = x^2$ , then $(f \circ g)(5) =$ ..............

(a) 2 (b) 4 (c) 22 (d) 25

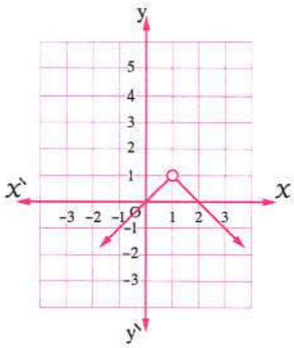
(10) The symmetric point on the curve of the function $f : f(x) = \frac{x+1}{x}$ is ..............

(a) (0 , 1) (b) (1 , 0) (c) (0 , 0) (d) (1 , $-1$)

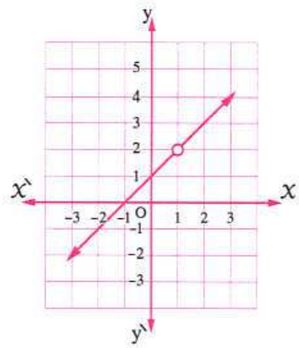
**(11)** $\lim\limits_{x \to 4} \dfrac{\sqrt{x}-2}{x-4} = \text{..............}$

(a) 2 (b) 4 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

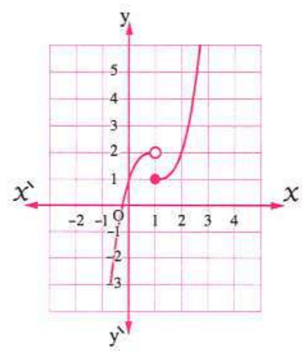
**(12)** Which of the following functions has no limit at $x = 1$ ?



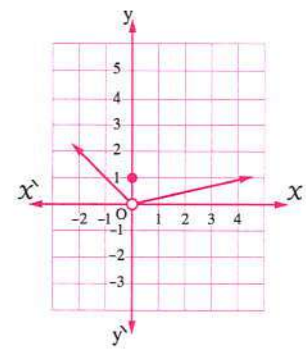

(a)




(b)




(c)




(d)

---

## 2 Answer the following questions :

**(1)** Graph the function $f : f(x) = \begin{cases} x^2 + 1 & , x > 0 \\ -x^2 - 1 & , x < 0 \end{cases}$

from the graph determine the range of the function and discuss its monotony. (2 marks)

**(2)** If $f(x) = x^2 - 3$ , $g(x) = \sqrt{x-2}$ , find $(f \circ g)(x)$ in the simplest form , determining its domain , then find $(f \circ g)(3)$ (2 marks)

**(3)** Find : $\lim\limits_{x \to 1} \dfrac{\sqrt{4x+5}-3}{x-1}$ (2 marks)

**(4)** ABCD is a parallelogram in which m (∠ A) = 50° , m (∠ DBC) = 70° and BD = 8 cm.

Find the perimeter of the parallelogram to the nearest whole number. (2 marks)

# Test 2

Total mark

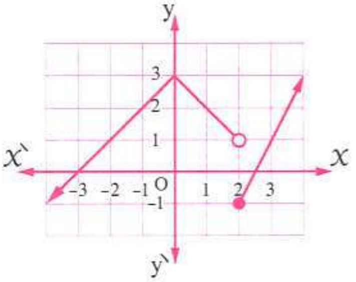
.............. / 20

## 1 Choose the correct answer from those given : (12 marks)

(1) In $\Delta ABC$ , $\frac{4\,b}{\sin B} = \ldots\ldots\ldots\ldots\; r$ , where r is the radius of the circle passes through the vertices of triangle ABC.

(a) 4 (b) 8 (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

(2) The opposite figure represents the curve of the function $f$ , then $\lim\limits_{x \to 2} f(x) = \ldots\ldots\ldots\ldots$




(a) 3

(b) 1

(c) – 1

(d) not exist.

(3) The one - to - one function from the functions defined by the following rules is ..............

(a) $f(x) = x^2$ (b) $f(x) = -2$ (c) $f(x) = |x|$ (d) $f(x) = \frac{1}{x}$

(4) If $f$ is an even function and the curve of the function passes through the point $(-3 , 2m + 1)$ and $f(3) = 5$ , then $m = \ldots\ldots\ldots\ldots$

(a) zero (b) – 1 (c) 1 (d) 2

(5) The domain of the function $f : f(x) = \sqrt{x - 3}$ is ..............

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $[3 , \infty[$ (d) $]-\infty , 3[$

(6) If $f(x) = \sqrt{x}$ , $g(x) = x^2 - 1$ , then $(f \circ g)(-3) = \ldots\ldots\ldots\ldots$

(a) 2 (b) – 4 (c) $2\sqrt{2}$ (d) undefined.

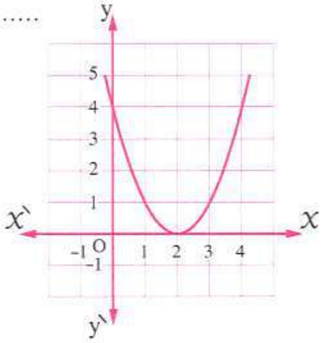
(7) $\lim\limits_{x \to 7} \frac{x^2 - 49}{7 - x} = \ldots\ldots\ldots\ldots$

(a) 14 (b) – 49 (c) 49 (d) – 14

(8) The point of symmetry of the function $f : f(x) = (x - 2)^3 + 1$ is ..............

(a) (2 , 1) (b) (2 , – 1) (c) (– 2 , 1) (d) (– 1 , – 2)

(9) The opposite figure represents the function $f : f(x) = \ldots\ldots\ldots\ldots$ where $f : \mathbb{R} \longrightarrow \mathbb{R}$




(a) $x^2 + 2$ (b) $(x + 2)^2$

(c) $x^2 - 4x + 4$ (d) $-(x - 2)^2$

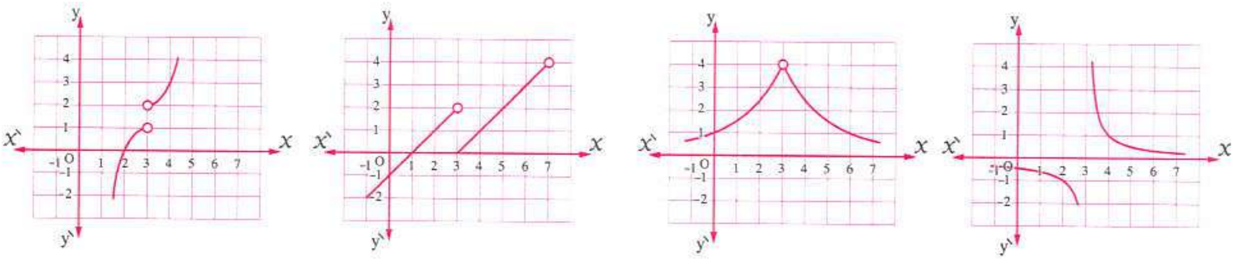
**(10)** If $\lim\limits_{x \to 3} \dfrac{x^2 - \ell}{x - 3} = m$ , then $(\ell , m) =$ ..............

(a) $(3 , 3)$ (b) $(-9 , 0)$ (c) $(9 , 6)$ (d) $(0 , 0)$

**(11)** In $\Delta$ XYZ , if m ($\angle$ X) : m ($\angle$ Y) : m ($\angle$ Z) = 2 : 3 : 1 , then $x : y : z =$ ..............

(a) $3 : \sqrt{2} : 1$ (b) $\sqrt{3} : 2 : 1$ (c) $\sqrt{2} : \sqrt{3} : 1$ (d) $2 : 3 : 1$

**(12)** Which of the following functions has a limit at $x = 3$ ?



(a) (b) (c) (d)

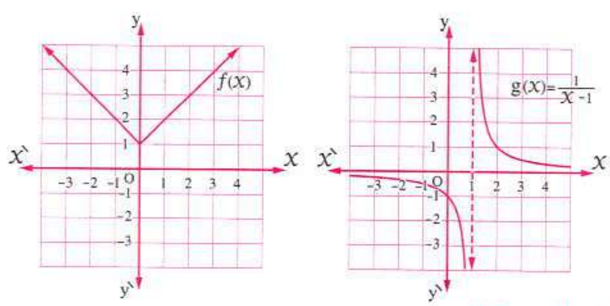
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## 2 Answer the following questions :

**( 1 )** Graph the curve of the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ , $f(x) = |x| + 1$. From the graph find the range and discuss the monotony and determine its type wheather it is odd , even or neither. (2 marks)

**( 2 ) In the opposite figure :**

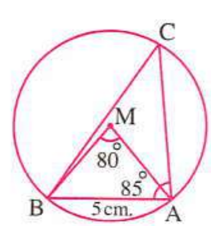
Find $(f \circ g)(x)$ and state its domain




(2 marks)

**( 3 ) Find :** $\lim\limits_{x \to 5} \dfrac{\sqrt{x + 11} - 4}{x^2 - 25}$ (2 marks)

**( 4 ) In the opposite figure :**




A circle M , AB = 5 cm. , m ($\angle$ AMB) = 80°

and m ($\angle$ CAB) = 85°

**Find : ( 1 )** The perimeter of $\Delta$ ABC.

**( 2 )** The area of the circle M. (2 marks)

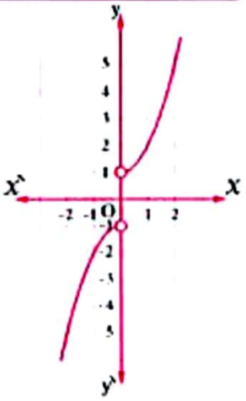
# Answers of October tests

## Answers of Test 1

### 1

| | | | |
|---|---|---|---|
| (1) b | (2) c | (3) c | (4) c |
| (5) c | (6) a | (7) d | (8) b |
| (9) c | (10) a | (11) d | (12) c |

### 2



(1) From the graph :

- The range of the function $f = \mathbb{R} - [-1, 1]$
- The function is increasing on $\mathbb{R} - \{0\}$

(2) $\because (f \circ g)(X) = f\left(g(X)\right) = f\left(\sqrt{X-2}\right)$

$= \left(\sqrt{X-2}\right)^2 - 3 = X - 2 - 3$

$= X - 5$

$, \because D_1 = \text{domain of } g = [2, \infty[$

, the values of $X$ which makes $g(X)$ in the domain of $f$ is $D_2 = \mathbb{R}$

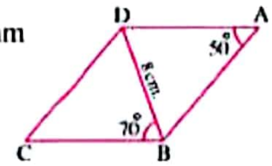
$\therefore$ The domain $(f \circ g) = D_1 \cap D_2 = [2, \infty[$

$, (f \circ g)(3) = 3 - 5 = -2$

(3) $\lim\limits_{X \to 1} \frac{\sqrt{4X+5} - 3}{X - 1} \times \frac{\sqrt{4X+5} + 3}{\sqrt{4X+5} + 3}$

$= \lim\limits_{X \to 1} \frac{4X + 5 - 9}{(X-1)\left(\sqrt{4X+5} + 3\right)}$

$= \lim\limits_{X \to 1} \frac{4(X-1)}{(X-1)\left(\sqrt{4X+5} + 3\right)} = \frac{4}{6} = \frac{2}{3}$



(4) $\because$ ABCD is a parallelogram

$\therefore m(\angle C) = 50°$

In $\Delta$ BDC :

$m(\angle BDC) = 180° - (50° + 70°) = 60°$

$\therefore \frac{8}{\sin 50°} = \frac{BC}{\sin 60°} = \frac{DC}{\sin 70°}$

$\therefore BC \simeq 9$ cm. , $DC \simeq 9.8$ cm.

$\therefore$ The perimeter of the parallelogram

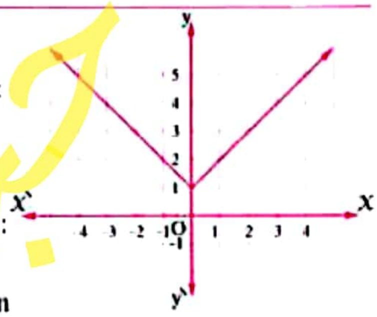
$= 2(BC + CD) \simeq 38$ cm.

## Answers of Test 2

### 1

| | | | |
|---|---|---|---|
| (1) b | (2) d | (3) d | (4) d |
| (5) c | (6) c | (7) d | (8) a |
| (9) c | (10) c | (11) b | (12) c |

### 2



(1) From the figure :

- The range $= [1, \infty[$
- The monotony : The function is decreasing on $]-\infty, 0[$ and increasing on $]0, \infty[$
- The type : Even.

(2) $f(X) = |X| + 1$ , $g(X) = \frac{1}{X-1}$

$\therefore (f \circ g)(X) = f[g(X)] = f\left(\frac{1}{X-1}\right)$

$= \left|\frac{1}{X-1}\right| + 1$

$\therefore D_1 = \text{domain of } g = \mathbb{R} - \{1\}$

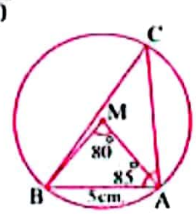
, the values of $X$ which makes $g(X)$ in the domain of $f$ is $D_2 = \mathbb{R}$

$\therefore$ Domain of $(f \circ g) = D_1 \cap D_2 = \mathbb{R} - \{1\}$

(3) $\lim\limits_{X \to 5} \frac{\sqrt{X+11} - 4}{X^2 - 25} \times \frac{\sqrt{X+11} + 4}{\sqrt{X+11} + 4}$

$= \lim\limits_{X \to 5} \frac{X + 11 - 16}{(X-5)(X+5)\left(\sqrt{X+11} + 4\right)}$

$= \lim\limits_{X \to 5} \frac{1}{(X+5)\left(\sqrt{X+11} + 4\right)} = \frac{1}{80}$



(4) In $\Delta$ ABC :

$m(\angle C) = \frac{1}{2} m(\angle AMB)$

$= 40°$

$\therefore m(\angle ABC) = 180° - (40° + 85°) = 55°$

$\therefore \frac{5}{\sin 40°} = \frac{b}{\sin 55°} = \frac{a}{\sin 85°} = 2r$

$= \frac{\text{Perimeter of } \Delta ABC}{\sin 40° + \sin 55° + \sin 85°}$

$\therefore$ Perimeter of $\Delta ABC \simeq 19.12$ cm.

$, r = 3.89$ cm.

$\therefore$ The area of the circle $M = \pi r^2 = \pi (3.89)^2$

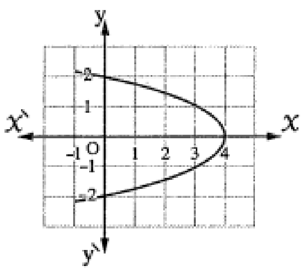
$\simeq 47.5 \text{ cm}^2$.

(Unit 1) Real functions

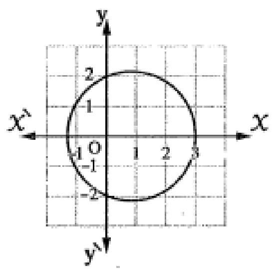
## (Domain, Range and monotony)

1-

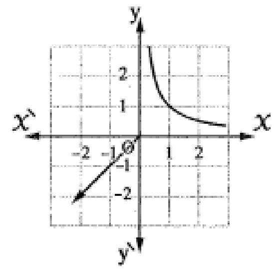
( **1** ) Which of the following figures represents a function of $x$ ?



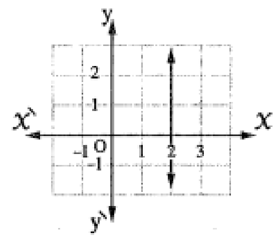

(a)




(b)

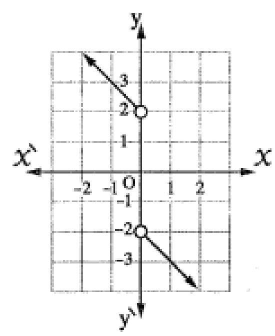



(c)




(d)

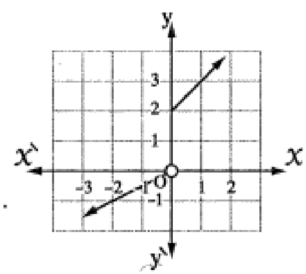
( **2** ) The opposite figure represents a function of $x$ whose domain is ...............




(a) $\mathbb{R}$ (b) $\mathbb{R} - ]-1 , 2[$

(c) $\mathbb{R} - [-1 , 2]$ (d) $\mathbb{R} - \{0\}$

( **3** ) The opposite figure represents a function of $x$ whose range is ...............




(a) $\mathbb{R} - [0 , 2]$ (b) $\mathbb{R} - \{0\}$

(c) $\mathbb{R} - [0 , 2[$ (d) $\mathbb{R} - ]0 , 2]$

( **4** ) $f(x) = \sqrt{4 - x^2}$ , then the domain of the function $f$ = ...............

(a) $[-2 , 2]$ (b) $]-2 , 2[$

(c) $[-2 , 2[$ (d) $]-2 , 2]$

# Operations on functions (composition functions)

2-

**Choose the correct answer from those given :**

**( 1 )** $f(x) = \frac{1}{x}$ , $g(x) = \sqrt{x}$ , then the domain of $(f.g) =$ ...............

(a) $\mathbb{R} - \{0\}$ (b) $\mathbb{R}$ (c) $\mathbb{R}^+$ (d) $[0, \infty[$

**( 2 )** $f(x) = x + 1$ , $g(x) = x^2$, then $(f \circ g)(2) =$ ...............

(a) 3 (b) 4 (c) 5 (d) 9

**( 3 )** The domain of the function $f : f(x) = \sqrt{5 - x}$ equals ...............

(a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R}^+$ (c) $]-\infty, 5]$ (d) $[5, \infty[$

**( 4 )** $f(x) = \sqrt{x}$ , $g(x) = x^2$ , then the domain of $(f \circ g) =$ ...............

(a) $[0, \infty[$ (b) $\mathbb{R}$ (c) $\mathbb{R}^+$ (d) $\mathbb{R}^-$

3-

If $f(x) = \frac{1}{x}$ , $g(x) = x + 3$ , find :

**( 1 )** $(f \circ g)(x)$

**( 2 )** $(g \circ f)(x)$

and state the domain in each case.

4-

If $f(x) = \frac{1}{x}$ , $g(x) = x + 3$ , find :

**( 1 )** $(f \circ g)(x)$

**( 2 )** $(g \circ f)(x)$

and state the domain in each case.

5-

If $f(\mathcal{X}) = \frac{1}{\mathcal{X}}$ , $g(\mathcal{X}) = 2\mathcal{X}$ , find each of the following :

(1) $(f + g)(\mathcal{X})$ (2) $(f.g)(\mathcal{X})$ (3) $\left(\frac{g}{f}\right)(\mathcal{X})$

Showing the domain.

# Properties of functions

6-

**Find the type of each function whether it is even , odd or otherwise :**

**( 1 )** $f(X) = \dfrac{X^3}{|X|+2}$ **( 2 )** $f(X) = \sin X^2 - \sin^2 X$

7-

**[b]** If $f(X) = X - 1$ , $g(X) = \sqrt{X}$ , then find $(g \circ f)(X)$ and determine its domain , then find $(g \circ f)(5)$

8-

From the following functions , the one-to-one function is ...............

(a) $f_1(X) = X + 2$ (b) $f_2(X) = X^2$ (c) $f_3(X) = |X|$ (d) $f_4(X) = 5$

9-

Find the domain of $f : f(X) = \dfrac{3X - 5}{X - 2}$ , then prove that $f$ is one-to-one.

10-

Draw the graph of the function $f : f(X) = \begin{cases} X|X| & \text{when } X < 0 \\ \dfrac{X^4}{|X|} & \text{when } X > 0 \end{cases}$

, then deduce its domain and discuss its type whether it is even , odd or otherwise.

# Graphical (basic and piecewise)

11-

Graph the function $f : f(x) = 4 - (x - 2)^2$ , then deduce its range , its monotony and whether the function is odd , even or otherwise.

12-

Graph the function $f : [-2 , 6] \longrightarrow \mathbb{R}$ where $f(x) = \begin{cases} 4 - x & , \quad -2 \leq x < 1 \\ x & , \quad 1 \leq x \leq 6 \end{cases}$

and from the graph deduce its range and discuss its monotonicity.

13-

Graph the function $f : f(x) = \begin{cases} x - 1 & , \quad 2 < x \leq 4 \\ -1 & , \quad -2 \leq x \leq 2 \end{cases}$

from the graph determine its range.

14-

**[b]** If $f : [-4 , 3] \longrightarrow \mathbb{R}$ , $f(x) = \begin{cases} 4 & \text{when } x < 0 \\ (x - 1)^2 + 1 & \text{when } 0 \leq x \leq 3 \end{cases}$

, graph the function $f$ , then from the graph , **deduce :**

( **1** ) The range.

( **2** ) The monotonicity.

( **3** ) The type (even , odd , otherwise).

## Geometric transformations

15-

Use the graph of the function $f$ where $f(X) = X^2$ to represent the function g where $g(X) = (X-1)^2 + 2$ and from the graph determine the range of the function g and discuss its monotonicity and tell whether it is even, odd or otherwise.

16-

Use the curve of the function $f$ where $f(X) = X^3$ to represent each of the following functions :

(1) $f_1(X) = (X+1)^3$ (2) $f_2(X) = X^3 + 1$

17-

If $f(X) = X^2 - 1$ , $g(X) = X + 1$

Graph the function $\frac{f}{g}$ , show its domain and its range.

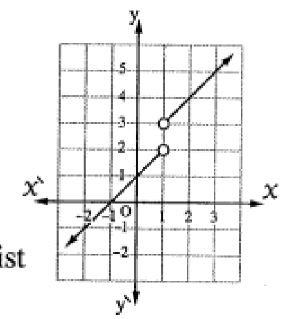
18-

Graph the curve of the function $f : f(X) = X|X|$ , then discuss its monotony and its type whether its even, odd or otherwise.

# (Unit 3 ) Introduction of limits of functions

1-

**Choose the correct answer from those given :**

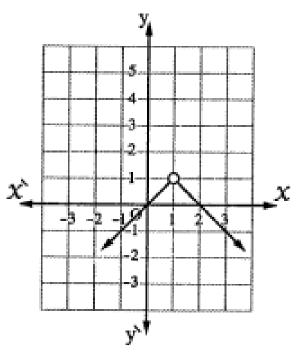


**( 1 )** The opposite figure represents the graph of the function $f$ , then

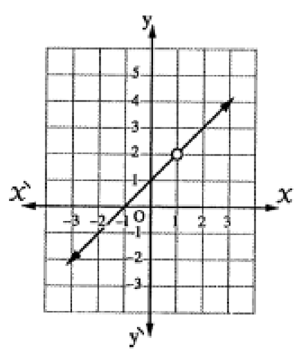
$\lim_{x \to 1} f(x) =$ ...............

(a) 2 (b) 3 (c) 1 (d) not exist

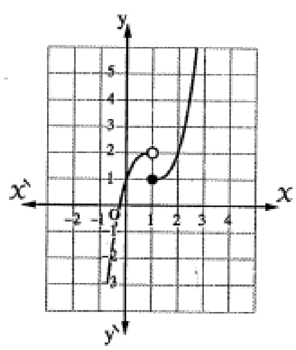
**( 2 )** Which of the following functions has no limit at $x = 1$ ?



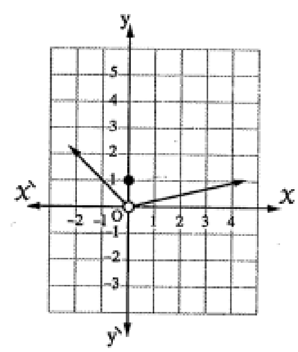
(a)



(b)



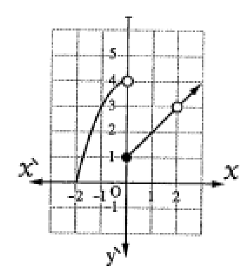
(c)



(d)

2-

**In the opposite figure , find :**



**( 1 )** $f$ (zero$^+$) **( 2 )** $f$ (zero$^-$)

**( 3 )** $f$ (2) **( 4 )** $\lim_{x \to 2} f(x)$

# Finding tke limit of the function algebrically

3-

**Choose the correct answer from those given :**

( 1 ) $\lim\limits_{x \to 0} \frac{1+x}{4x-1} =$ ...............

(a) $-1$ (b) $\frac{1}{4}$ (c) $-\frac{1}{4}$ (d) 1

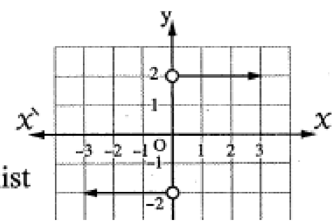
( 2 ) $\lim\limits_{x \to 3} \frac{x^2-9}{x-3} =$ ...............

(a) $-6$ (b) zero (c) 3 (d) 6

( 3 ) The opposite figure represents $f(x)$

Then : $\lim\limits_{x \to 2} f(x) =$ ...............

(a) 0 (b) $-2$ (c) 2 (d) not exist




4-

**Find :**

$$\lim_{x \to 3} \frac{x^2-x-6}{x-3}$$

5-

**Find :**

$$\lim_{x \to -1} \left(\frac{5x^2+5x}{3x^2-3}\right)$$

6-

$\lim\limits_{x \to 0} \frac{x^2+x}{x} =$ ..........

7-

**Find :** $\lim\limits_{x \to 1} \frac{x^3-2x^2+1}{x^2+x-2}$

8-

**Find :** (2) $\lim\limits_{x \to 2} \dfrac{x^2 - 5x + 6}{x^3 - 8}$

9-

**Find :** $\lim\limits_{x \to 5} \dfrac{x - 5}{2x - 3}$

10-

**Find :** $\lim\limits_{x \to 2} \dfrac{x^3 + 8}{x + 2}$

11-

$\lim\limits_{x \to -3} \dfrac{x^2 + 4x + 3}{x^2 - 9}$

12-

**Find :** (1) $\lim\limits_{x \to -2} \dfrac{x^3 + 8}{x + 2}$

## (Unit 4) The sine rule

1-

Solve $\Delta ABC$ in which $m(\angle B) = 35^\circ$ , $m(\angle C) = 70^\circ$ , and the diameter length of its circumcircle = 32 cm.

2-

In $\Delta ABC$ , if $m(\angle A) = 35^\circ$ , $a = 17$ cm. and $b = 20$ cm.

**Prove that :** $\Delta ABC$ has two solutions , then find them.

3-

Find the perimeter of $\Delta ABC$ in which $m(\angle A) = 57^\circ\ 13'$ , $c = 8.7$ cm. and $m(\angle B) = 64^\circ\ 18'$

4-

ABC is a triangle in which : $m(\angle A) = 35^\circ$ , $a = 8$ cm. and $b = 6$ cm. **Find :** $m(\angle B)$

5-

ABC is a triangle in which : $b = 12$ cm. , $m(\angle B) = 75^\circ$ and $m(\angle C) = 45^\circ$ **Find :**

( **1** ) a

( **2** ) The area of $\Delta ABC$

( **3** ) The radius length of the circumcircle of the triangle ABC

6-

ABC is a triangle in which m (∠ A) : m (∠ B) : m (∠ C) = 3 : 4 : 3

If a = 5 cm. , find the perimeter of Δ ABC

7-

Solve the triangle ABC in which a = 8 cm. , m (∠ A) = 60° and m (∠ B) = 40°

8-

Solve the acute-angled triangle ABC in which a = 21 cm. , b = 25 cm. and the diameter length of its circumcircle = 28 cm.

9-

Find the shortest side length in Δ ABC , in which : m (∠ A) = 43° , m (∠ B) = 70° and c = 9 cm. Find the area of the triangle ABC.

10-

ABC is a triangle in which : AC = 4.7 cm. , m (∠ B) = 34° and m (∠ C) = 66°

Find the length of $\overline{BC}$ , then find the area of its circumcircle.

11-

ABC is a triangle in which m (∠ A) = 40° , a = 5 cm. and b = 7 cm.

Find m (∠ B) approximating to the nearest minute.

12-

Solve the triangle ABC in which a = 5 cm. , b = 7 cm. and m (∠ C) = 65°